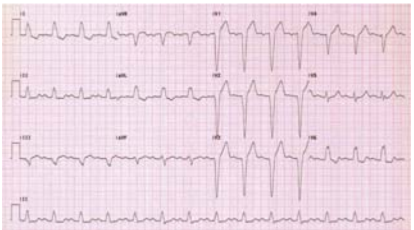
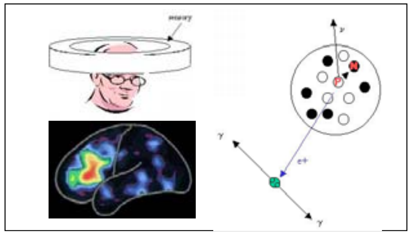
Medical signal processing



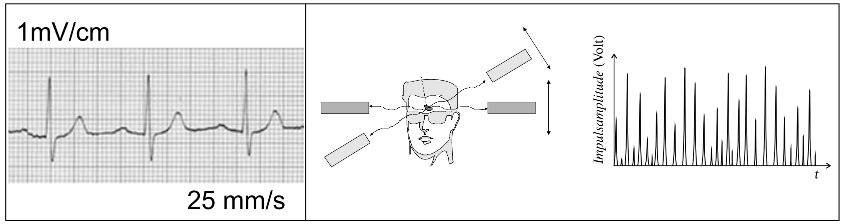


A **signal** is any kind of physical quantity that conveys/ transmits/stores information

e.g. (1)
electrical voltage, that can be measured on the surface of the skin/head as a result of the heart-/muscle-/brain activities (ECG/EMG/EEG)

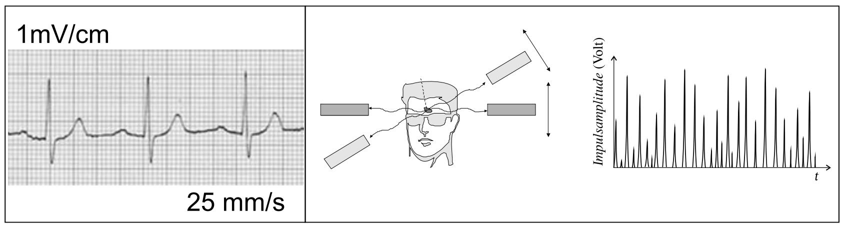
e.g. (2)
gamma photon detection in radioisotope diagnostics

(1) (2)



Classification of signals

static	–	time-dependent
periodic	–	non-periodic
random	–	deterministic
pulsed	–	continuous
electric	–	non-electric
analog	–	digital



in a very special role

electric signals

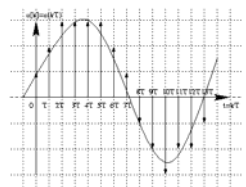
non-electric signals are transferred to electric ones

advantages of **electric** signals: they are easy to transform, amplify, transmit

digital signals

analog signals are transferred to digital ones

advantages of **digital** signals: they are easy to store, the noise can be engineered and influence can be reduced



quantity that compares the magnitudes of two signals:

Signal level or **Bel**-number (or **Decibel**-number): n

(named after A. Bell)

unit of n : Bel (B) or decibel (dB)

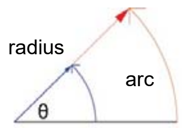
$$n = \lg\frac{P_2}{P_1}\,\mathrm{B} = \lg\frac{J_2}{J_1}\,\mathrm{B} = \lg\frac{E_2}{E_1}\,\mathrm{B}$$

decimal logarithm of ratio of two powers (intensities, energies)

cf. **radian**

$$\Theta = \frac{\text{arc}}{\text{radius}}$$

$$[\Theta] = \frac{\mathrm{m}}{\mathrm{m}} = \mathrm{rad} = 1$$



cf. **pH** (power of Hydrogen)

$$\mathrm{pH} = -\lg\frac{[\mathrm{H}^+]}{1\mathrm{M}}$$

$$\textit{e.g.}:\ [\mathrm{H}^+] = 10^{-7}\mathrm{M}$$

$$\Rightarrow \mathrm{pH} = -\lg 10^{-7} = -1\cdot(-7) = 7$$

---

instead of Bel number we are using **decibel-number**

$$\boxed{n = 10\cdot\lg\frac{P_2}{P_1}\,\mathrm{dB}}$$

(10d = 1)

the **characteristic** unit: **power** (or intensity/energy),
the **practical** unit: (electric) **voltage**

the relation between power and voltage:

$$P = U\cdot I = \frac{U^2}{R} \quad (\mathrm{Ohm}:\ U = R\cdot I)$$

signal level with voltages:

$$n = 10\cdot\lg\frac{P_2}{P_1}\,\mathrm{dB} = 10\cdot\lg\frac{\frac{U_2^{\,2}}{R_2}}{\frac{U_1^{\,2}}{R_1}}\,\mathrm{dB} \overset{R_2\approx R_1}{=}$$

$$= 10\cdot\lg\frac{U_2^{\,2}}{U_1^{\,2}}\,\mathrm{dB} = \boxed{20\cdot\lg\frac{U_2}{U_1}}\,\mathrm{dB}$$

$$\frac{P_2}{P_1} = 2 \Leftrightarrow 10\lg 2\,\mathrm{dB} =$$
$$= 10\cdot 0{,}3\,\mathrm{dB} = 3\mathrm{dB}$$

$$\frac{P_2}{P_1} = \frac{1}{2} \Leftrightarrow -3\mathrm{dB}$$

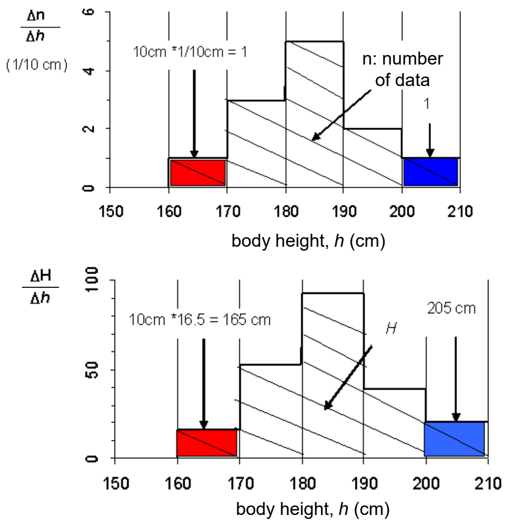
cf. half life,
half value thickness

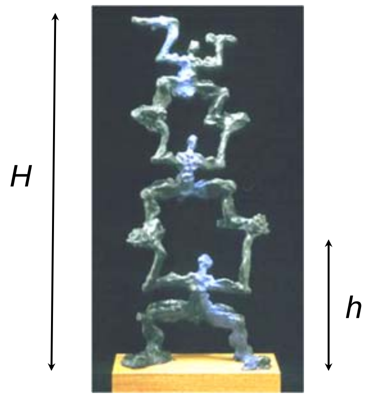
$$\frac{P_2}{P_1} = 10 \Leftrightarrow 10\cdot\lg 10\,\mathrm{dB} =$$
$$= 10\cdot 1\mathrm{dB} = 10\mathrm{dB}$$

$$\frac{P_2}{P_1} = 100 \Leftrightarrow 10\lg 100\,\mathrm{dB} =$$
$$= 10\cdot 2\mathrm{dB} = 20\mathrm{dB}$$

U_2/U_1	P_2/P_1	dB
1,414	2	3
2	4	6
	8	9
3,16	10	10
	20	13
10	100	20
	$1000=10^3$	30
$100=10^2$	$10000=10^4$	40
$1000=10^3$	10^6	60

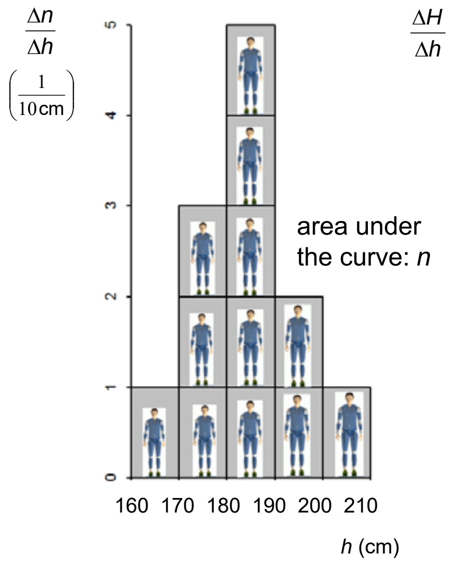
empirical density function



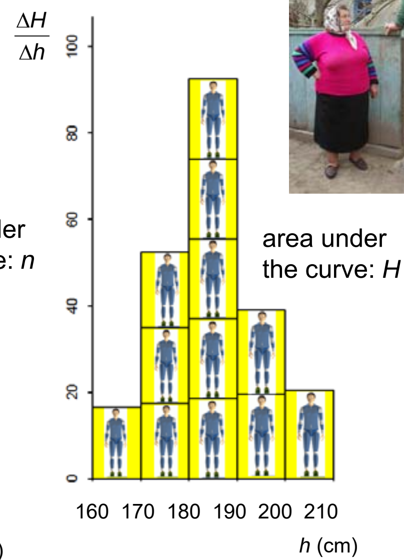


spectrum, as a special density function

Density function



Spectrum



Fourier's theorem for periodic functions (signals)

all (usual) periodic functions can be expressed as a sum of sine (and cosine) functions from the fundamental frequency and the overtones

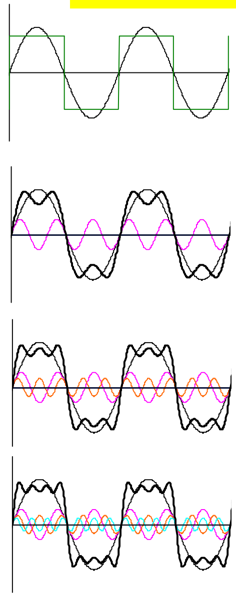
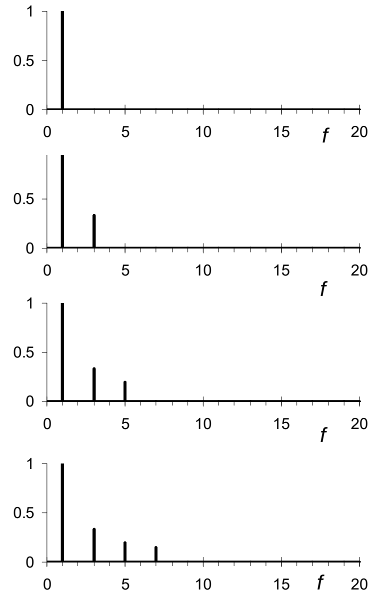
periodic function:
there is a period, T

$\frac{1}{T} = f$, where f is the frequency

the sine function, which has the same frequency as the periodic function:
fundamental frequency — in music: pitch

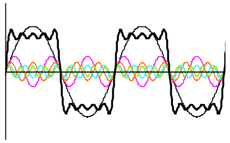
$2f$, $3f$, $4f$, ... : **overtones** — in music: timbre/tone color

(line spectrum)

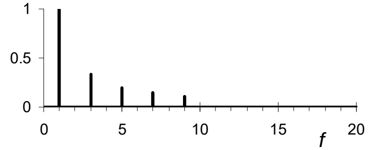
function		spectrum
	square pulse train fundamental fr(equency) fundamental fr.+ 3rd overtone fundamental fr.+ 3rd overtone + 5th overtone fundamental fr.+ 3rd overtone + 5th overtone + 7th overtone	

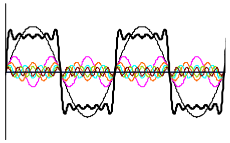
function

spectrum

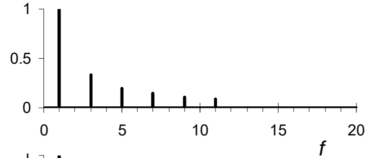


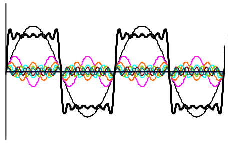
fundamental fr.+
3rd overtone +
+ ... +
9th overtone



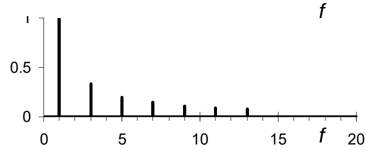


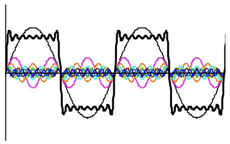
fundamental fr.+
3rd overtone +
+ ... +
11th overtone



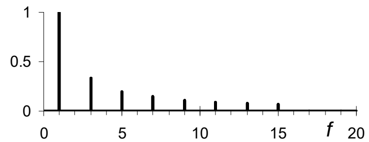


fundamental fr.+
3rd overtone +
+ ... +
13th overtone



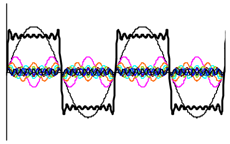


fundamental fr.+
3rd overtone +
+ ... +
15th overtone

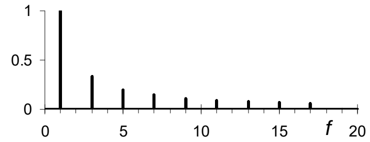


function

spectrum

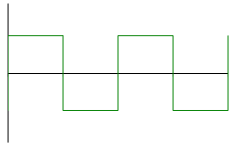


fundamental fr.+
3rd overtone +
+ ... +
17th overtone

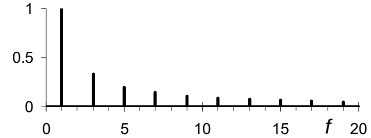


...

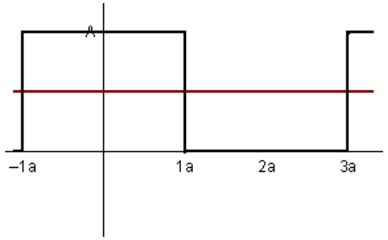
...



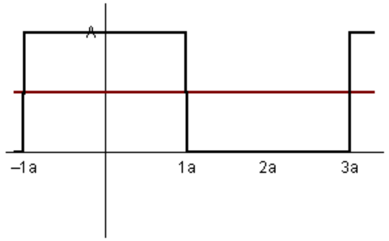
fundamental fr.+
3rd overtone +
+ ... +
17th overtone +
+ ...



individual summands up to order 0

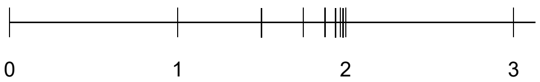


summation



cf. infinite **series**

$$\sum_{k=0}^{\infty}\frac{1}{2^k}=\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots=2$$



$$\sum_{k=0}^{31}\frac{1}{2^k}=\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^{31}}=2,\quad \text{error}: 4{,}6\cdot 10^{-10}$$

sufficiently accurate(?)

$$\sum_{k=0}^{49}\frac{1}{2^k}=\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^{49}}=2,\quad \text{error}: 3{,}55\cdot 10^{-15}$$

sufficiently accurate

i	$1/2^k$	$\Sigma(1/2^k)$	$\varepsilon=2-\Sigma(1/2^k)$
0	1	1	1
1	0,5	1,5	0,5
2	0,25	1,75	0,25
3	0,125	1,875	0,125
4	0,0625	1,9375	0,0625
5	0,03125	1,96875	0,03125
6	0,015625	1,984375	0,015625
7	0,007813	1,9921875	0,0078125
8	0,003906	1,99609375	0,00390625
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26	1,49E-08	1,999999985	1,49012E-08
27	7,45E-09	1,999999993	7,45058E-09
28	3,73E-09	1,999999996	3,72529E-09
29	1,86E-09	1,999999998	1,86265E-09
30	9,31E-10	1,999999999	9,31323E-10
31	4,66E-10	2	4,65661E-10
32	2,33E-10	2	2,32831E-10
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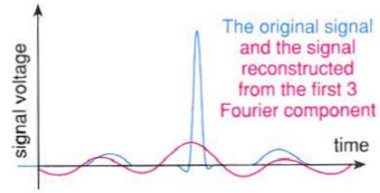
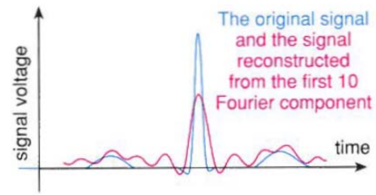
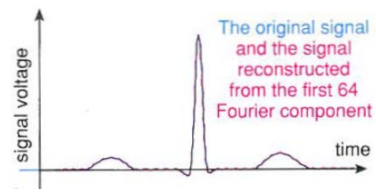
[illegible]	[illegible]	2	[illegible]
47	7,11E-15	2	7,10543E-15
48	3,55E-15	2	3,55271E-15
49	1,78E-15	2	0
50	8,88E-16	2	0

Creating an ECG signal from sine functions



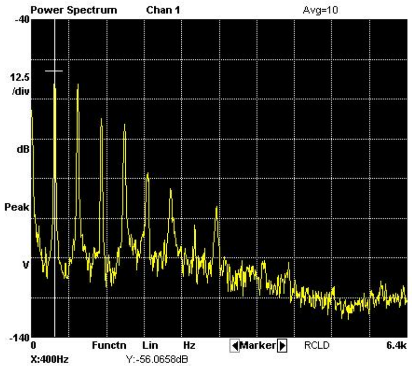


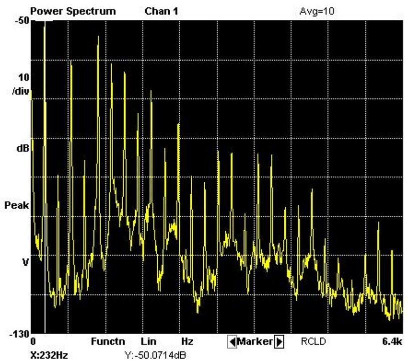


Textbook, Figure VII.3.

Measured spectra

flute | clarinet



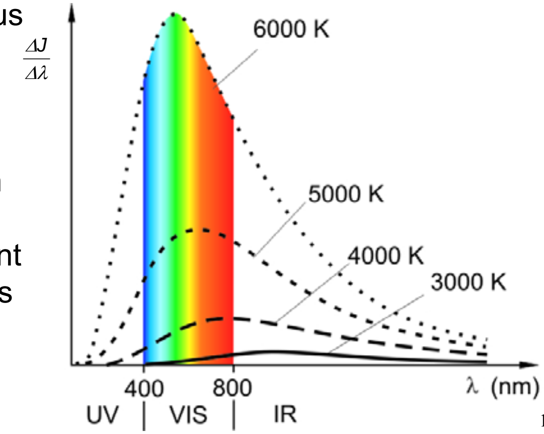


Fourier'theorem for non-periodic functions (signals)

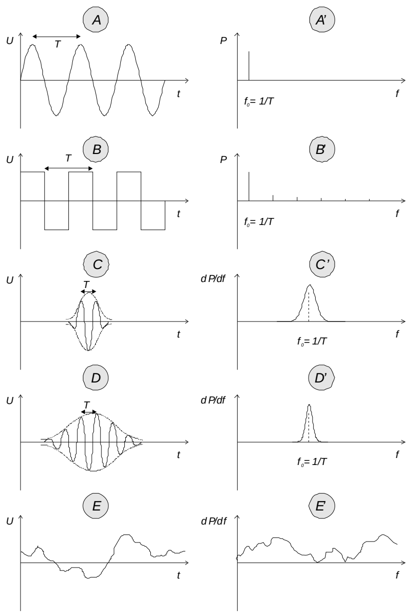
all (usual) functions can be expressed as a sum of sine (and cosine) functions

spectrum: continuous

cf. emission spectra of incandescent light sources



function | spectrum



sine function — line spectrum (1 line)

periodic function — line spectrum

a few periods — band spectrum

eg. pulsed ultrasound

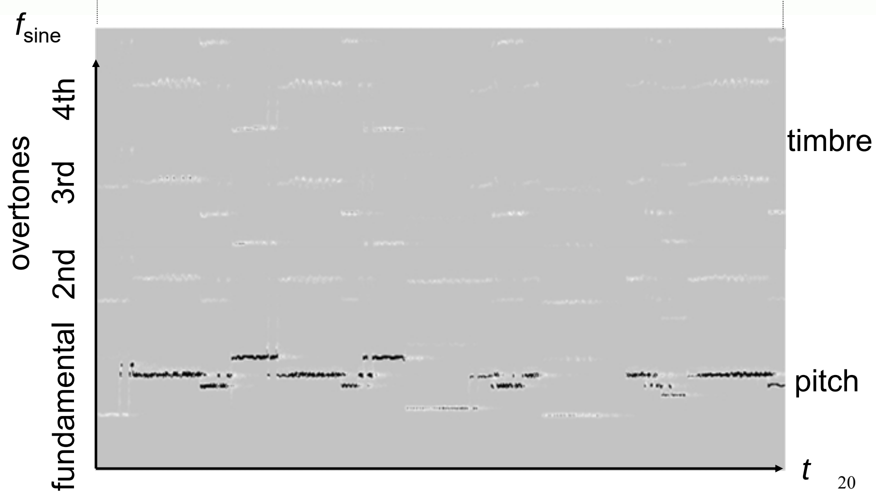
more periods — band spectrum

non-periodic function — continuous spectrum

Music in time-frequency representation

Inisheer

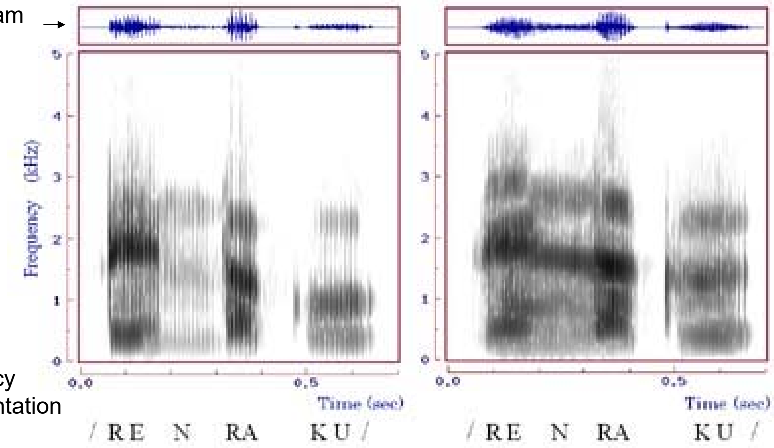
Traditional



Voiceprint

oscillogram (sound intensity vs. time)

time – frequency representation

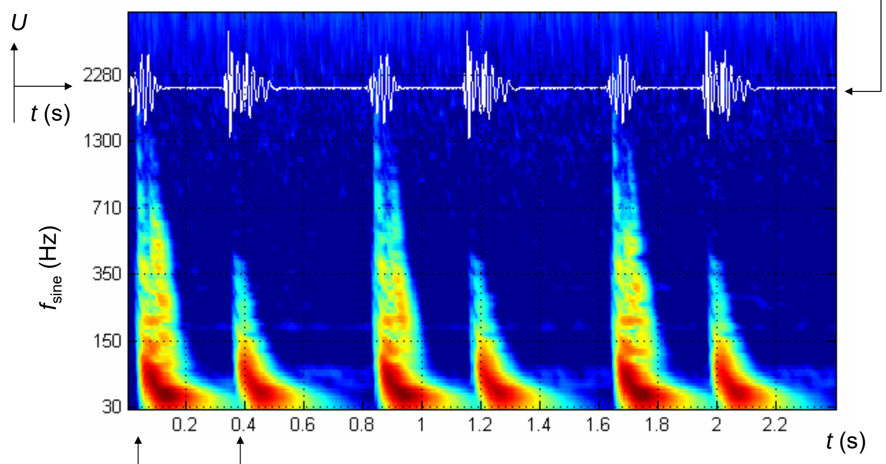


pronounced consonants and vowels

http://www.nrips.go.jp/org/fourth/info3/index-e.html

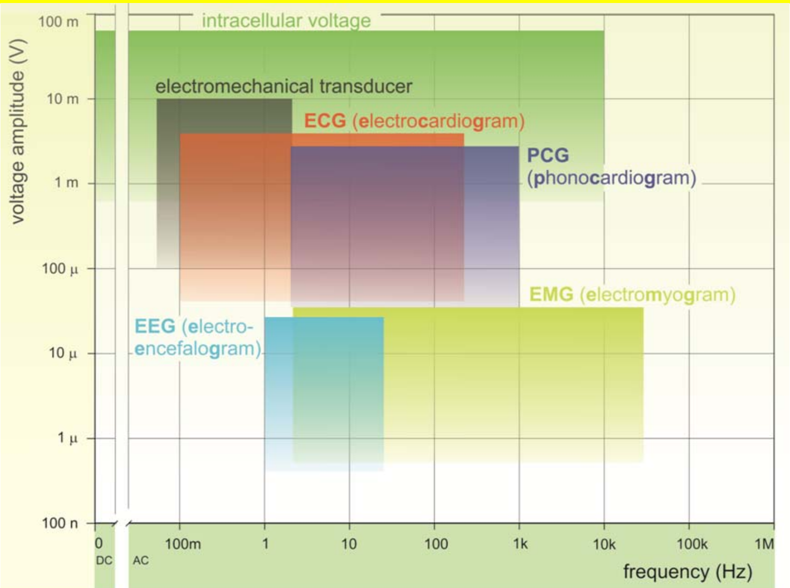
Heart beats in time-frequency representation

(+ oscillogram)



systole diastole

Frequency and amplitude ranges of biological signals



Practical manual, titel page of meas. 17

Frequency dependent unit: Electronic amplifier

(1) $P_{in} < P_{out}$

(2) P_{in} and P_{out} : same functions

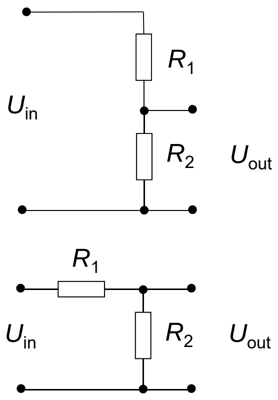
same: „fundamentalist“ requirement
similar: realistic requirement

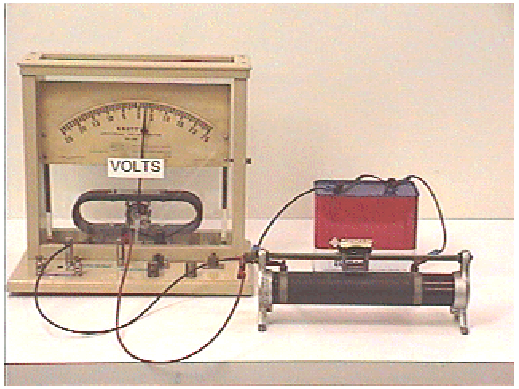
$(1)+(2) \quad A_P \cdot P_{in}(t) \equiv P_{out}(t)$, where $A_P > 1$

$\boxed{A_P = \frac{P_{out}}{P_{in}},}$ **power gain** (amplification)

$\boxed{A_U = \frac{U_{out}}{U_{in}},}$ **voltage gain** (amplification)

(frequency independent) voltage-divider





$$U_{out} = \frac{R_2}{R_1 + R_2} U_{in}$$

frequency dependent voltage-divider: with capacitor

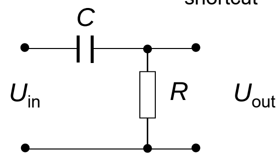
supplementary material

High-pass/low-cut **filter**

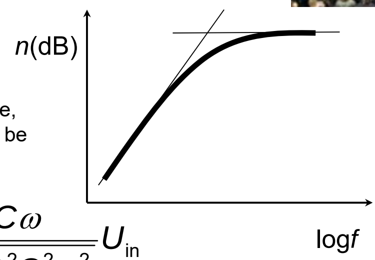
stray/parasitic capacitance

$$R_C = \frac{1}{C\omega}$$

at high frequencies the capacitor is a shortcut



because of the phase difference, the sum should be calculated as vectors



$$U_{out} = \frac{R}{\sqrt{\frac{1}{C^2\omega^2} + R^2}} U_{in} = \frac{RC\omega}{\sqrt{1 + R^2C^2\omega^2}} U_{in}$$

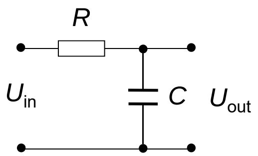
at very low frequencies: if $\omega << \omega_0$ $(\omega \approx 0), U_{out} = 0$

at low frequencies: if $\omega << \omega_0, U_{out} = RC\omega\, U_{in}$ ↔ 6 dB/octave

at high frequencies : if $\omega \approx \infty, U_{out} = U_{in}$

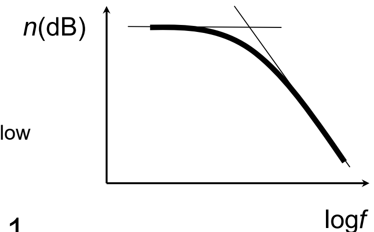
supplementary material

Low-pass/high-cut **filter**



$$R_C = \frac{1}{C\omega}$$

the capacitor at low frequencies is a discontinuity



$$U_{out} = \frac{\frac{1}{C\omega}}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} U_{in} = \frac{1}{\sqrt{R^2C^2\omega^2 + 1}} U_{in}$$

at low frequencies: if $\omega << \omega_0$ $(\omega \approx 0), U_{out} = U_{in}$

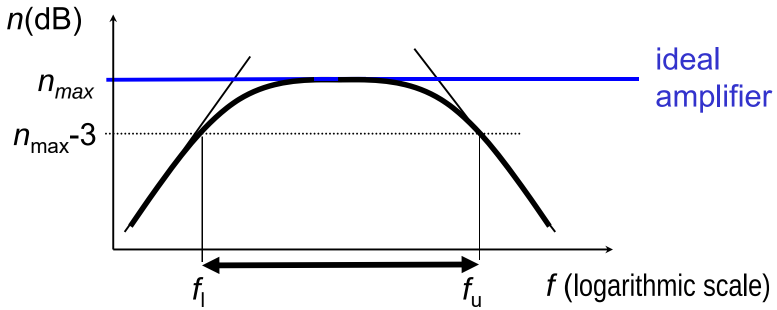
at high frequencies: if $\omega >> \omega_0, U_{out} = \frac{1}{RC\omega} U_{in}$ ↔ -6 dB/octave

at very high frequencies : if $\omega >> \omega_0$ $(\omega \approx \infty), U_{out} = 0$

for (1 [on page 24]): $A_P > 1$,

$n = 10 \lg A_P = 20 \lg A_U > 0$ dB

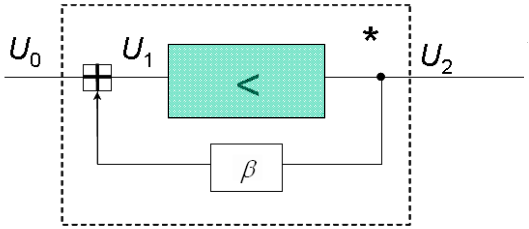
for (2 [on page 24]): **frequency characteristics**

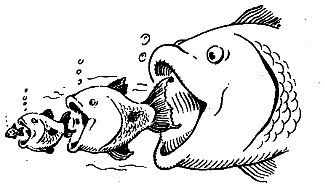


f_l : lower frequency limit

f_u : upper frequency limit

Amplifier with feedback





Without appropriate perspective we may not recognize our real position.

$(a)\quad U_1 = U_0 + \beta U_2 \qquad (b)\quad A_U = \frac{U_2}{U_1}$

$$(c)\quad A_U^* = \frac{U_2}{U_0} = \frac{U_1 A_U}{U_0} = \frac{(U_0 + \beta U_2) A_U}{U_0} = A_U + \beta \frac{U_2}{U_0} A_U = A_U + \beta A_U^* A_U$$

$$A_U^* - \beta A_U^* A_U = A_U \qquad \boxed{A_U^* = \frac{A_U}{1 - \beta A_U}}$$

$\boxed{A_U^* = \frac{A_U}{1 - \beta A_U},}$ A_U^* : voltage gain with feedback

A_U : voltage gain without feedback

$\beta > 0$, **positiv feedback** (same phase), $A_U^* > A_U$ (advantage)

$\beta < 0$, **negativ feedback** (in opposite phase), $A_U^* < A_U$ (disadv.)

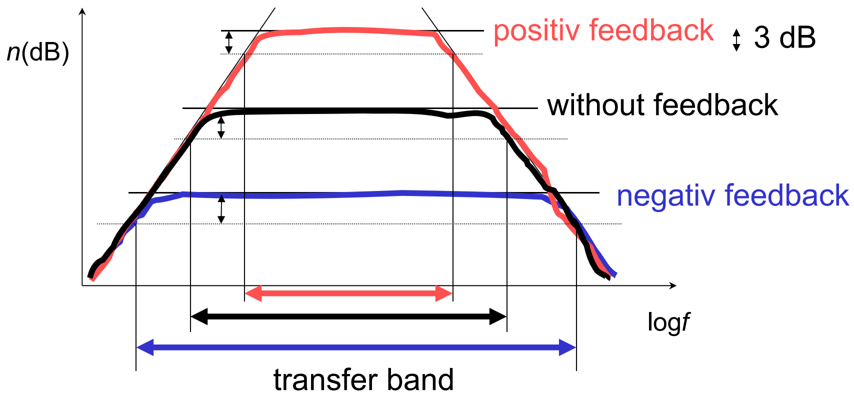
positiv feedback:

(a) $\beta A_U = 1$, amplification: „infinite“
– sine wave oscillator
e.g.: ultrasound generator, heat theraphy

(b) $\beta A_U \leq 1$, amplification: very big
– regenerative amplifier
e.g.: hearing, outer haircells

"... and whose idea was it to ask for employee feedback?"

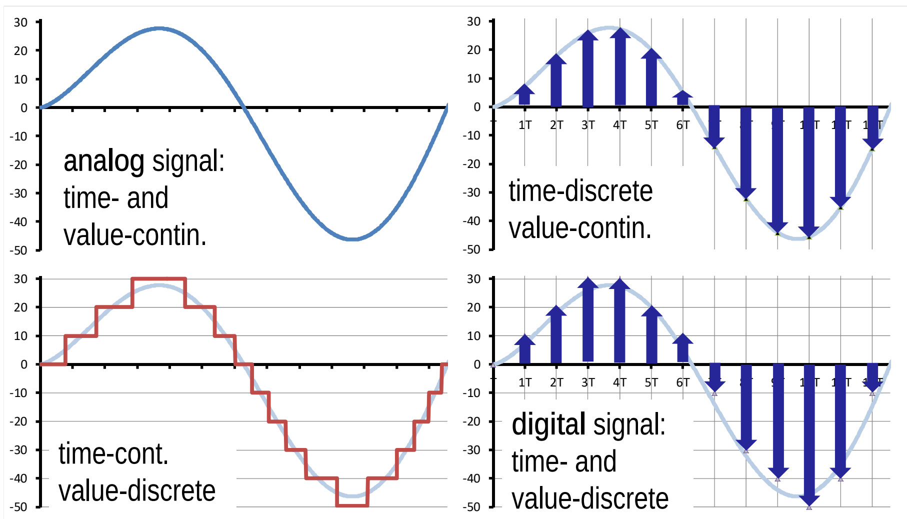
negativ feedback: „all“ amplifier



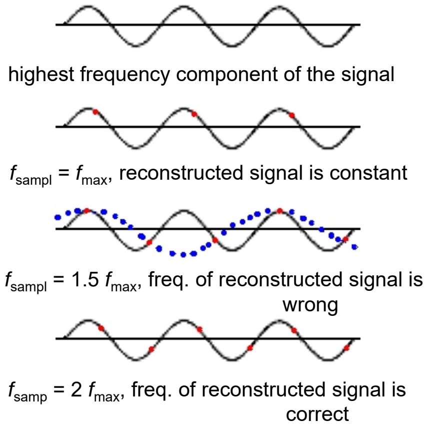
positiv feedback: transfer band – narrrower (big disadvantage)
higher gain (advantage)

negativ feedback: transfer band – broader (advantage)
less gain (small disadvantage)

Analog signal – digital signal



time-discrete: the value of the signal is not known for all moments in time



Nyquist–Shannon sampling theorem:

for complete reconstruction the minimum sampling frequency should be twice the frequency of the highest overtone of the signal

e.g.: hifi, $f_{max} = 20$ kHz

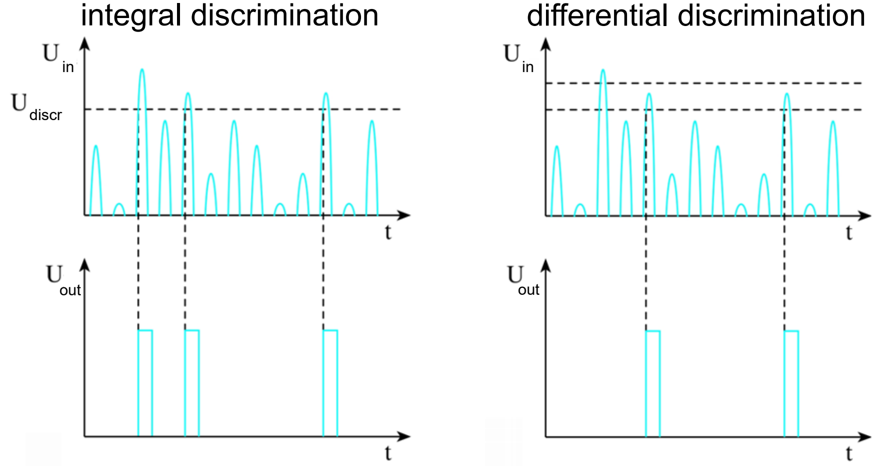
$f_{sampl} = 44.1$ kHz > 2*20 kHz

value discrete: the value of the signal can not be arbitrary

e.g.: hifi, 16 bit = 2^{16} = 65 536 (CD standard)

24 bit = 2^{24} = 16 777 216 (“best” audio card)

Pulse processing



to select only those pulses that are larger than a preset amplitude

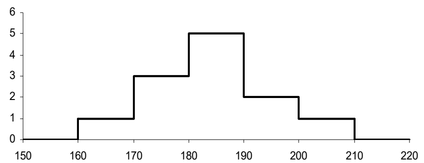
to select only those pulses whose amplitudes lie within a preset window

Textbook, Figure VII.32.

Distribution functions and ID/DD ”spectra”

$\frac{\Delta N}{\Delta h}\left(\frac{1}{10\,\text{cm}}\right)$

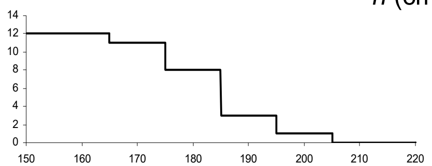
density function



DD-”spectrum”

$M=N_0-N$

”cumulative-distribution-function”

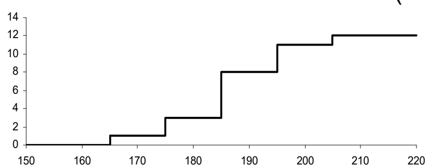


ID-”spectrum”

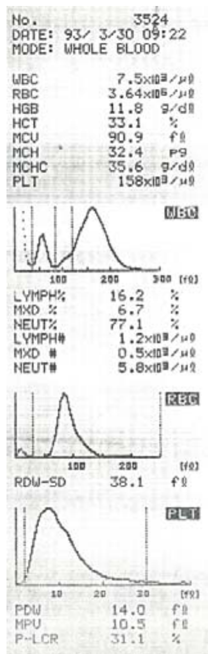
how many pulses are larger than *h*?

N

cumulative-distribution-function

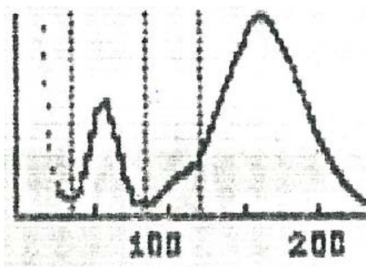


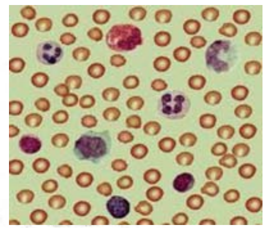
how many pulses are smaller than *h*?



Concentration of white blood cells

Coulter counter





LYMPH%	16.2	%
MXD %	6.7	%
NEUT%	77.1	%
LYMPH#	1.2x10³	/μℓ
MXD #	0.5x10³	/μℓ
NEUT#	5.8x10³	/μℓ