

The mysterious X-ray

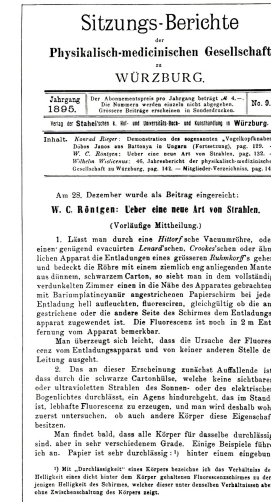
X-ray

Generation, properties, applications

Miklós Kellermayer



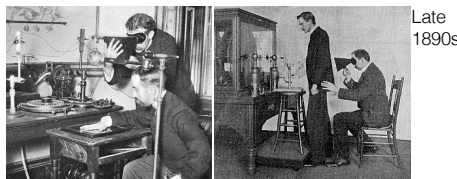
Wilhelm Konrad
Röntgen
(1845-1923)
Nobel prize, 1901



Hand mit Ringen (Hand with Ring): Wilhelm Röntgen's first "medical" X-ray, of his wife's, Anna Bertha Ludwig's hand, taken on 22 December 1895 and presented to Professor Ludwig Zehender of the Physik Institut (University of Freiburg, 1 January 1896).

Glorious history of x-ray

Transparency –
paper funnel
radioscope

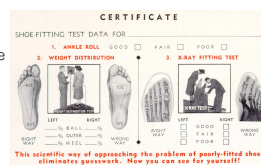


Late
1890s

World
war I.



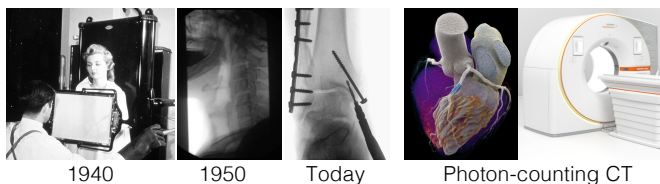
Everyday
applications



Airport
security



Medical
applications



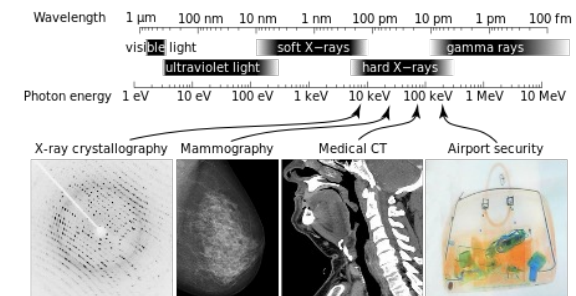
1940

1950

Today

Photon-counting CT

X-rays are electromagnetic waves



Wavelength 10 - 0.01 nm.

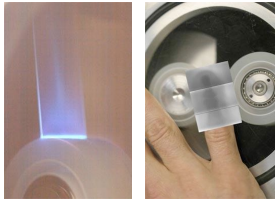
Frequency 30×10^{15} - 30×10^{18} Hz (petahertz - exahertz).

Energy 120 eV - 120 keV.

Generation of X-ray

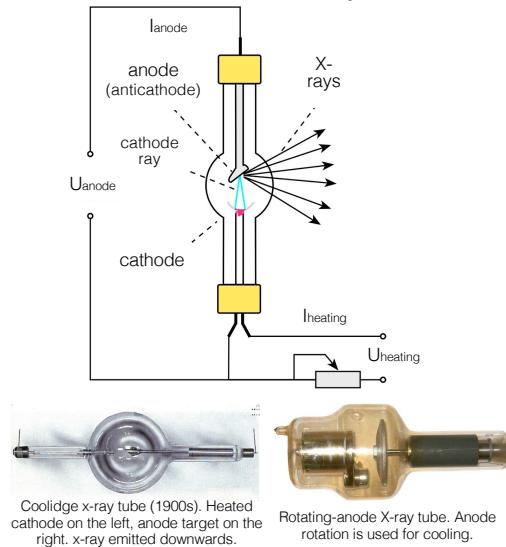
Non-conventional method

Triboluminescence: light emission evoked by scratching or rubbing. (Francis Bacon, 1605)



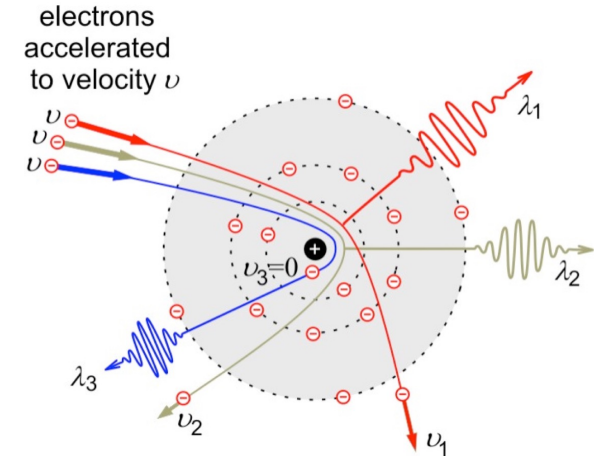
Peeling away sticky tape emits light and X-rays. (Nature News, October 2008)

Usual method: x-ray tube



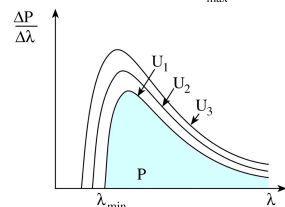
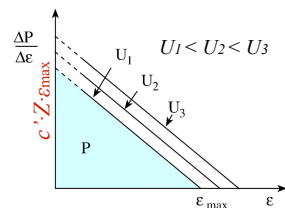
Mechanism I. "Bremsstrahlung"

Electrons **decelerate**, thereby loose their kinetic energy, when interacting with the atoms of the anode ("braking radiation").



Spectrum of Bremsstrahlung

Continuous



$$eU_{anode} = \epsilon_{max} = hf_{max}$$

Maximal photon energy (ϵ_{max})
N.B.: Total kinetic energy of electron is transformed in one step (rare event).
 e : electron's charge;
 U_{anode} : accelerating voltage;
 eU_{anode} : acceleration work;
 h : Planck's constant;
 f_{max} : limiting frequency

$$\lambda_{min} = \frac{hc}{e \cdot U_{anode}}$$

Limiting wavelength (λ_{min})
(Duane-Hunt Law)
N.B.: Limiting wavelength is inversely proportional to accelerating voltage.
 c : light speed;
 hc/e : constant (1.2398 kV·nm)

$$\frac{\Delta P}{\Delta \epsilon} = c \cdot Z \cdot (\epsilon_{max} - \epsilon)$$

Energy spectrum
(energy dependence of power)

$$P_{tot} = \frac{1}{2} c \cdot Z \cdot \epsilon_{max}^2 = c \cdot Z \cdot U_{anode}^2 \cdot e^2$$

$$P_{tot} = C_{Rtg} \cdot I_{anode} \cdot U_{anode}^2 \cdot Z$$

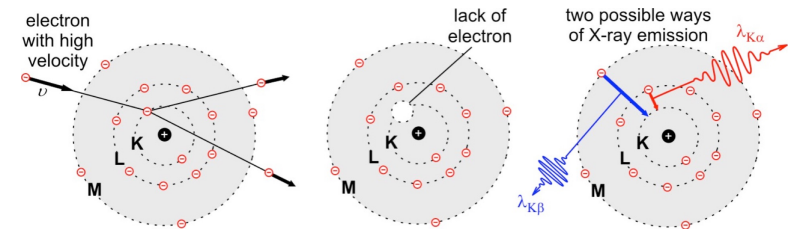
Total power (P_{tot})
(based on the area of the triangle)
 C_{Rtg} : coefficient ($1.1 \times 10^{-9} \text{ V}^{-1}$);
 I_{anode} : anode current (number of electrons hitting the anode per unit time);
 Z : atomic number of the anode atoms

$$\eta = \frac{P_{tot}}{P_{in}} = \frac{C_{Rtg} \cdot I_{anode} \cdot U_{anode}^2 \cdot Z}{I_{anode} \cdot U_{anode}} = C_{Rtg} \cdot U_{anode} \cdot Z$$

Efficiency (η)
 P_{in} : invested power
N.B.: Typically, $\eta < 1\%$.

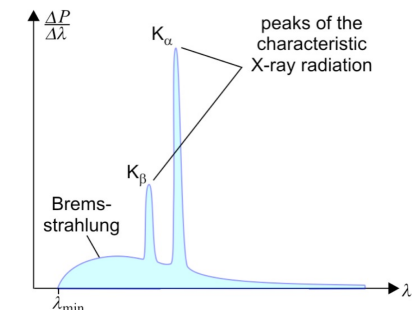
Mechanism II. Characteristic X-ray

Knocked-out inner-shell electron is replaced by one on a higher-energy shell



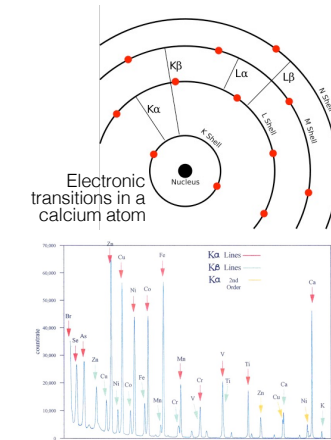
Spectrum of characteristic X-ray

Linear

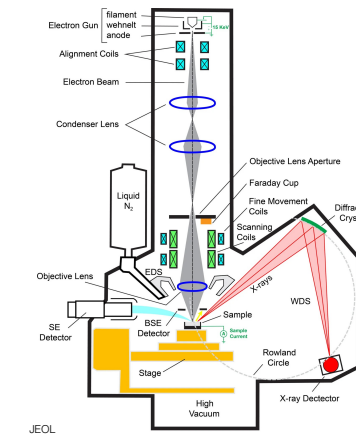


X-ray spectrum characterizes *atomic* composition

Because inner-shell electrons participate in characteristic X-radiation, only the *atomic* (and not the molecular) properties are revealed



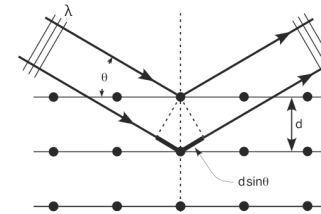
Energy dispersive X-ray fluorescence spectrum



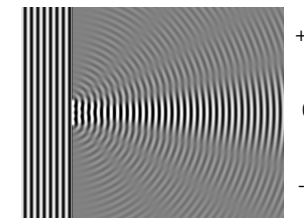
X-ray spectroscope (in an electron microscope!) (measures x-ray energy spectrum)

Interaction of x-ray with matter

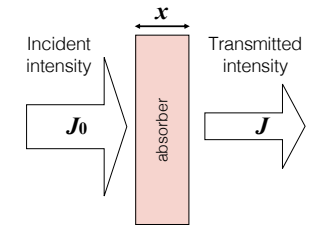
1. Diffraction



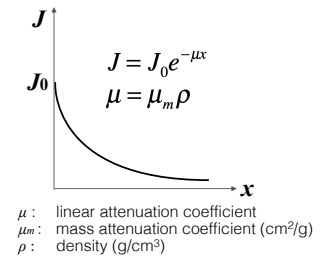
Condition of constructive interference: $2d \sin \theta = n\lambda$



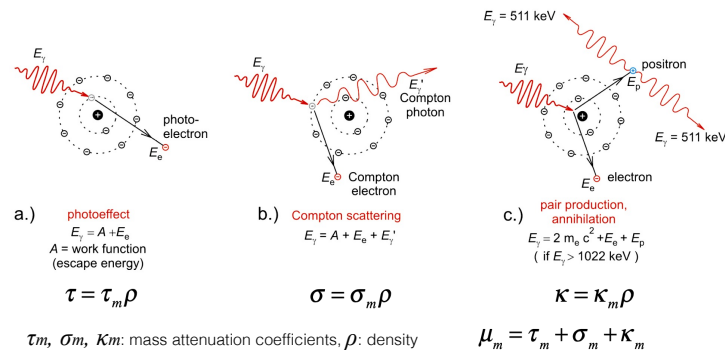
2. Absorption



Exponential **attenuation** principle



Attenuation mechanisms



a.) **photoeffect**
 $E_i = A + E_e$
 A = work function (escape energy)

$$\tau = \tau_m \rho$$

τ_m , σ_m , κ_m : mass attenuation coefficients, ρ : density

b.) **Compton scattering**
 $E_i = A + E_e + E_e'$

$$\sigma = \sigma_m \rho$$

c.) **pair production, annihilation**
 $E_i = 2 m_0 c^2 + E_e + E_p$
(if $E_i > 1022 \text{ keV}$)

$$\kappa = \kappa_m \rho$$

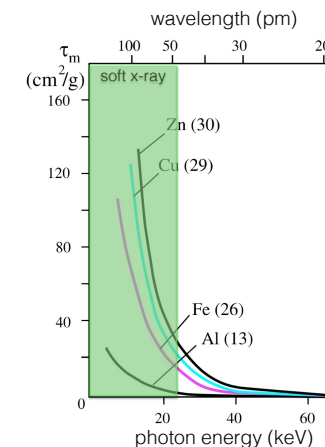
$$\mu_m = \tau_m + \sigma_m + \kappa_m$$

Mechanism	Photon energy (ϵ) dependence of the mass attenuation coefficient	Atomic number (Z) dependence of the mass attenuation coefficient	Relevant energy range in soft tissue
Photoeffect	$\sim 1 / \epsilon^3$	$\sim Z^3$	10 - 100 keV
Compton scatter	falls gradually with ϵ	$\sim Z/A$ (A : mass number)	0.5 - 5 MeV
Pair production	rises slowly with ϵ	$\sim Z^2$	> 5 MeV

Diagnostic X-ray:

1. Contrast mechanism between soft tissue and bone: photoeffect ($\sim Z^3$)
2. Contrast mechanism within soft tissue: Compton-scatter ($\sim \rho$)

Photoeffect attenuation depends strongly on photon energy and atomic number



$$\tau_m = \text{const} \cdot \frac{Z^3}{\epsilon^3} = C \cdot \lambda^3 \cdot Z^3$$

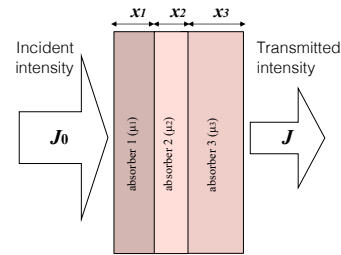
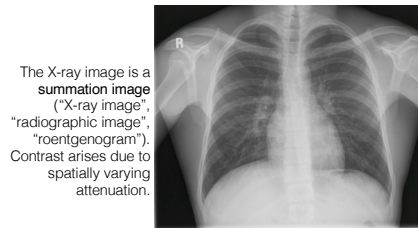
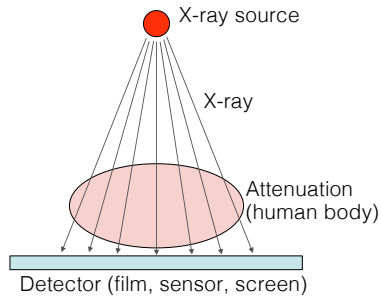
For multi-component system: "effective atomic number" (Z_{eff})

$$Z_{\text{eff}} = \sqrt[3]{\sum_{i=1}^n w_i Z_i^3}$$

ϵ =photon energy
 Z =atomic number
 w =mole fraction
 n =number of components

Material	Z_{eff}
Air	7.3
Water	7.7
Soft tissue	7.4
Bone	13.8

Application I. X-ray imaging



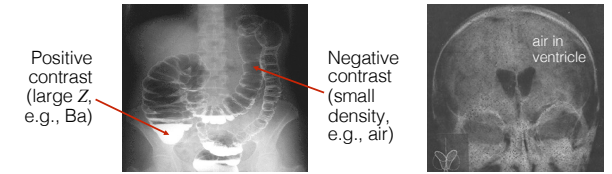
$$J = J_0 e^{-(\mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3 + \dots)}$$

$$\lg \frac{J_0}{J} = (\mu_1 x_1 + \mu_2 x_2 + \mu_3 x_3 + \dots) \cdot \lg e$$

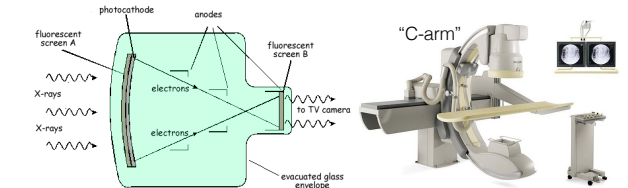
μ_n : n^{th} absorber's attenuation coefficient
 x_n : n^{th} absorber's thickness

Improving X-ray imaging I.

Increasing contrast:
contrast agents



Enhancing sensitivity:
intensifier

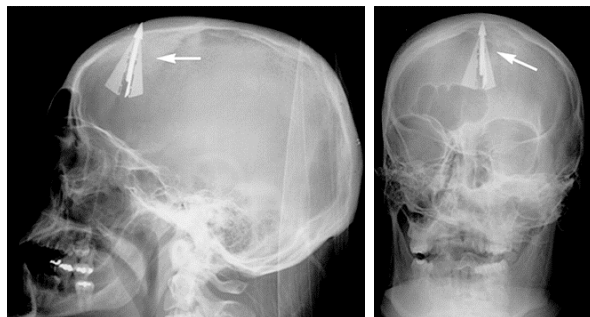


Background subtraction:
"Digital Subtraction Angiography" (DSA)



Improving X-ray imaging II. Spatial resolution

Bi-directional X-ray imaging

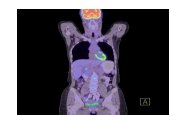
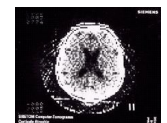
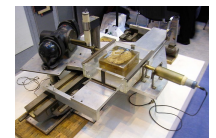
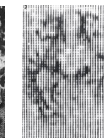


Bi-directional cranial X-ray of an individual who tried to commit suicide with a crossbow.

Improving X-ray imaging: the CAT scanner

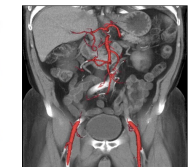
History

- Röntgen, Hounsfield and Cormack
- 1967: first CAT scan
- 1972: prototype
- 1974: first clinical CAT image (head)
- 1976: whole body CAT scan
- 1979: Nobel-prize
- 1990: spiral CAT scanner
- 1992: multislice CAT scanner
- 2006: 64 slice (and more...)
- multiple and hybrid modes: SPECT-CT, PET-CT, Dual-source CT



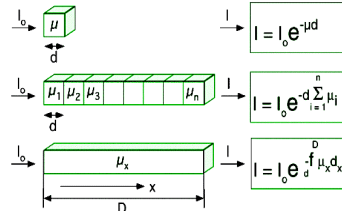
Summary

- **Tomographic** digital imaging method that uses **x-rays**: displays x-ray **absorbance** by the different points of the tomographic slice.
- **Multidetector** spiral CT (4-64 detector array): one slice 0.4-1 s; entire examination 5-15 s.
- **Ionizing** radiation. Absorbed **dose** ~50-100 times that of conventional x-ray. Significant **scattered** intensity.



CT Foundations I: determination of μ

Objective: to determine the attenuation coefficient (μ_x) of the individual volume elements (voxels)

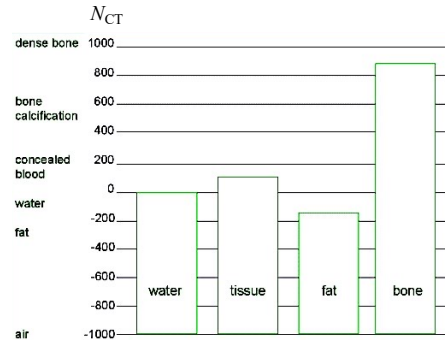


μ_x : linear attenuation coefficient
 dx : size of the voxel

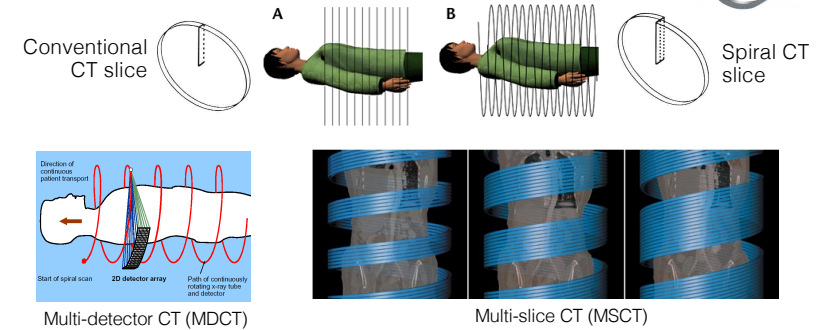
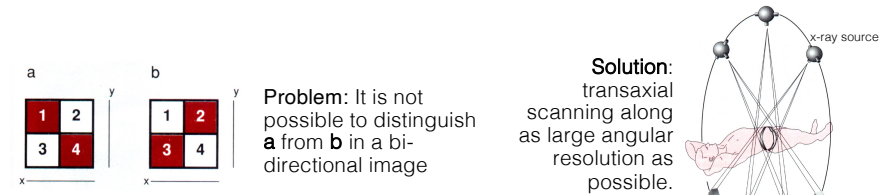
CT Image: density matrix

$$N_{CT} = 1000 \frac{\mu - \mu_w}{\mu_w}$$

N_{CT} : density, CT number, Hounsfield units
 μ : attenuation coefficient of voxel
 μ_w : attenuation coefficient of water

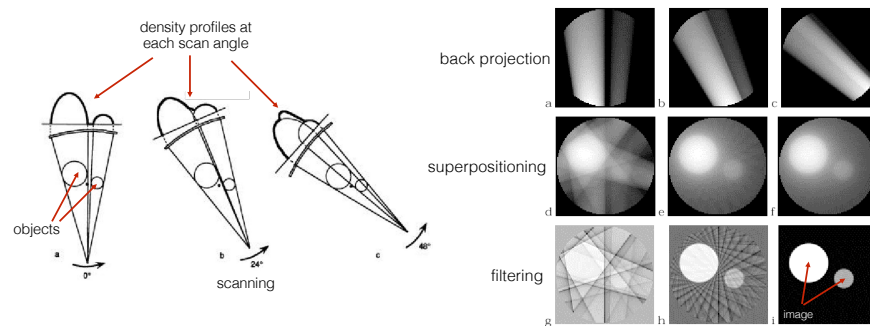


CT foundations II. scanning

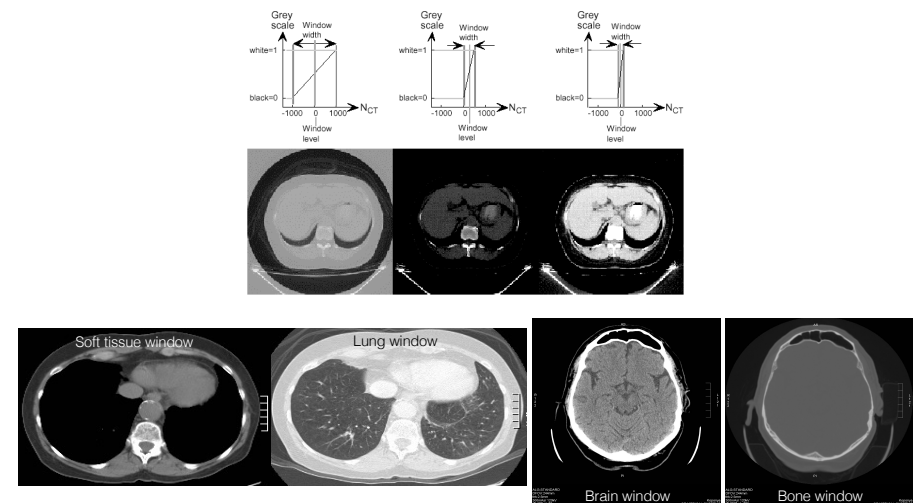


CT foundations III: Image Reconstruction

1. Algebraic reconstruction techniques
2. Direct Fourier reconstruction
3. „Filtered Back Projection” (current method)

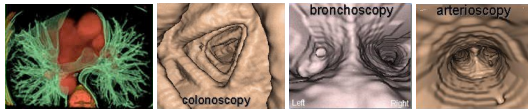


CT foundations IV: Contrast manipulation „Windowing”

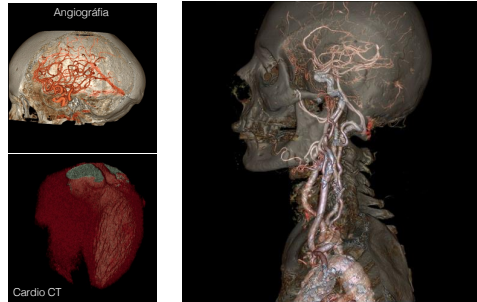


Modern CAT scanning

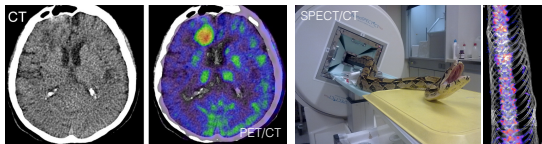
3D reconstruction,
Virtual
endoscopy



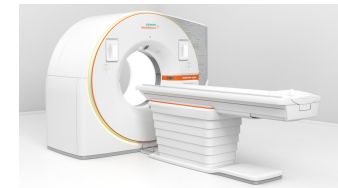
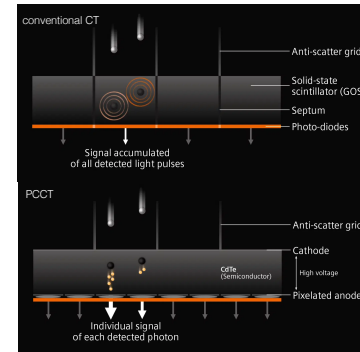
Increasing speed
and resolution



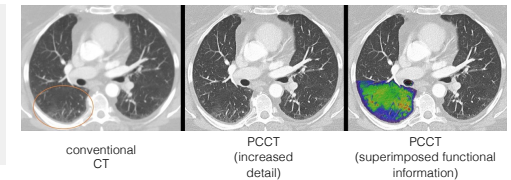
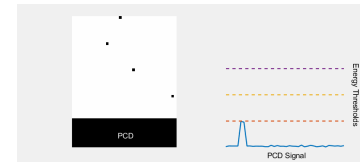
Combination with
other modalities



Photon Counting CT (PCCT)

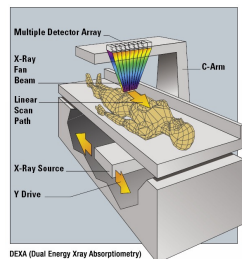


- PCD: Photon Counting Detector (cadmium telluride crystal, CdTe)
- PCD keeps track of the energy of incoming photons
- PCD provides x-ray energy spectrum
- increased sensitivity (lower x-ray dose, lower contrast-agent dose)
- functional imaging possibility

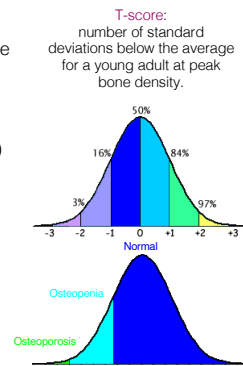
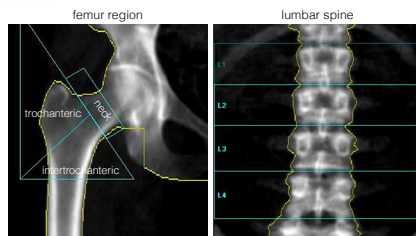


Application II. Absorptiometry

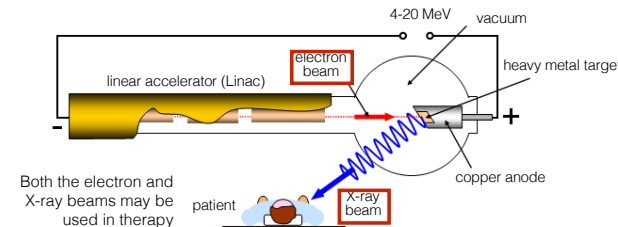
Dual-energy X-ray absorptiometry (DXA or DEXA)



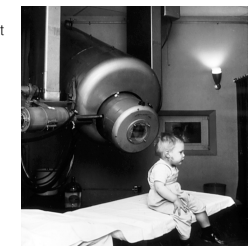
- Most important method for measuring bone density
- Characteristic X-ray is used as source
- Two different photon energies are employed (so that bone vs. soft-tissue absorption can be differentiated)
- Low dose is applied
- Whole-body scan is recorded
- Densities of distinct areas (e.g., femur, spine) are compared with reference databases
- Bone Mineral Density (BMD) calculated
- T-score is established



Application III. Radiation therapy



Modern hospital Linac



First patient (Gordon Isaacs) treated with Linac radiation therapy (electron beam) for retinoblastoma (1955)

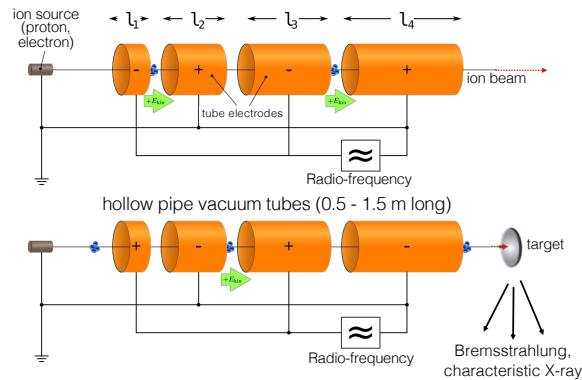
Advantages:

- Radiation may be turned on and off
- No contaminating radioactivity

Generating high-energy X-ray

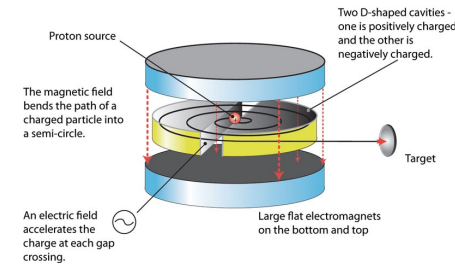
Linear accelerator (Linac)

- Charged particle (electron, proton) accelerated between electrodes (but not inside the electrode).
- Velocity of particle increases in steps.
- Electrode polarity is alternating.
- Electrodes are gradually longer (l_n increases) in order to maintain synchrony.
- Accelerated particles are directed at suitable target material (to generate X-ray).



Ring-shape particle accelerators

Cyclotron

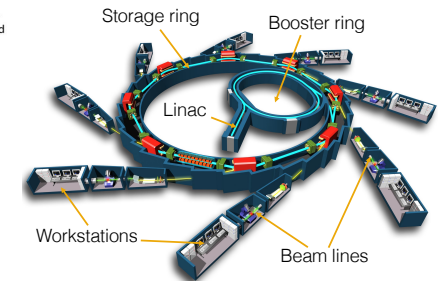


- Lorentz forces keep particles on circular path (causes limitations)
- Few tens of MeV particles are generated
- Used for generating positron-emitting isotopes (PET)
- Clinical cyclotrons in PET centers

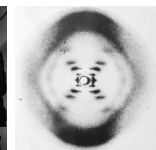


11 MeV medical cyclotron

Synchrotron



- Very high energy particles can be generated (GeV)
- Relativistic speeds can be achieved (near light speed)
- X-rays used for high-resolution structural research
- Few facilities around the world (Grenoble, Chicago, etc.)



J.D. Watson and C.F. Crick, and the first x-ray image of DNA (1953)

Feedback



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