Transport

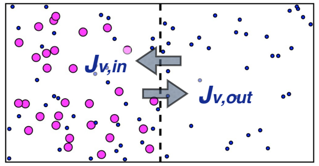
Osmosis, flow

Miklós Kellermayer

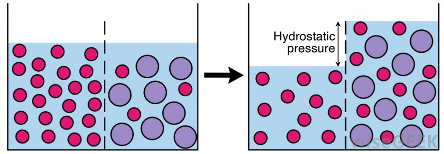
A special case of diffusion: osmosis

One-directional ***solvent*** transport by diffusion

Driving forces



Initial state | Final state



$J_{v,in}$ driving force: solvent concentration difference

$J_{v,out}$ driving force: pressure difference

Osmotic equilibrium: $$J_{v,in} = J_{v,out}$$

Osmotic pressure: $$p_{osmosis} = cRT$$

van't Hoff's Law:
c = concentration of solute
R = universal gas constant
T = absolute temperature

Importance of osmosis: cell swelling, edema, hemodialysis.

Mechanisms of osmosis

van't Hoff's gas-law mechanism

Universal gas law: $$pV = RT$$

$$p = \frac{1}{V}RT$$

Osmotic pressure: $$p_{osm} = \pi = cRT$$

p = pressure
V = volume
R = gas consant (8,3 J/mol.K)
T = absolute temperature
c = molar concentration of solute

Example:
What is the osmotic pressure of a 0.1 M (0.1 mol/ dm3) sucrose soution?

π = 8.3 (J/mol.K) x 293 (K) x 0.1 (mol/dm3) = 243 kPa ~ 2.4 atm.

Note: the formula applies to dilute solutions.

Importance of osmosis

Properties of the semipermeable membrane: ***reflection coefficient*** (σ)

For a perfect semipermeable membrane:

$$\Delta P = \Delta\pi \quad \text{and} \quad \sigma = \frac{\Delta P}{\Delta\pi} = 1$$

In reality: $$\Delta P < \Delta\pi \quad \text{and} \quad \sigma = \frac{\Delta P}{\Delta\pi} < 1$$

$$0 < \sigma < 1$$

Osmotic work:

$$-L = nRT\ln\frac{c_1}{c_2} = nRT\ln\frac{\pi_1}{\pi_2}$$

R = gas constant
n = total number of moles of solute
c_1 = initial molarity of solution
c_2 = final molarity of solution (after equilibration)
π_1 = initial osmotic pressure
π_2 = final osmotic pressure (after equilibration)

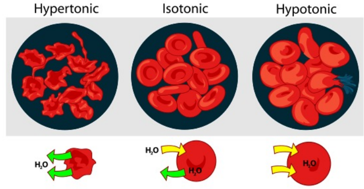
Equivalent osmotic pressure (osmotic concentration): concentration of non-electrolyte solution in equilibrium with a heterogenous solution system.

Units: mmol/kg = mOsmol/kg = mOsm

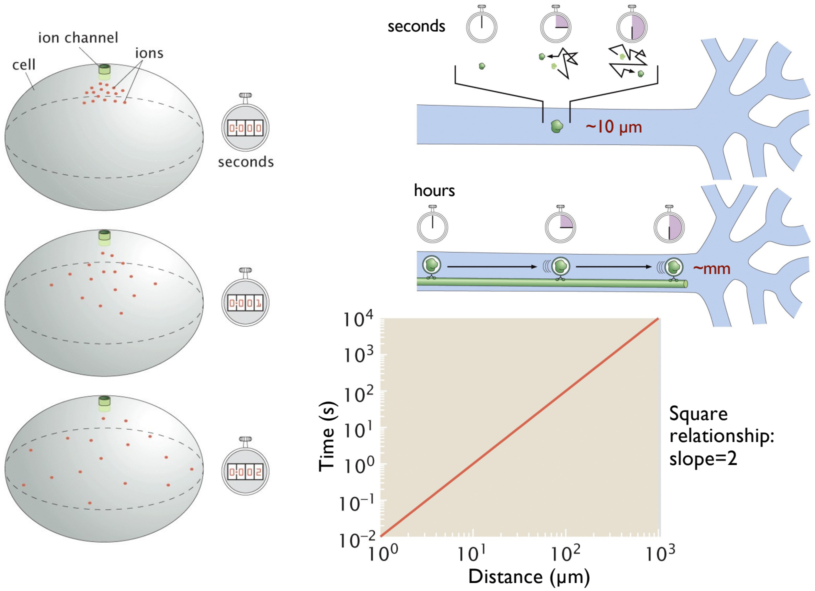
Osmolarity of blood plasma ~ 300 mOsm.

Oncotic pressure: colloid osmotic pressure. Osmotic pressure of a solution of colloidal macromolecules.

Hypertonic | Isotonic | Hypotonic



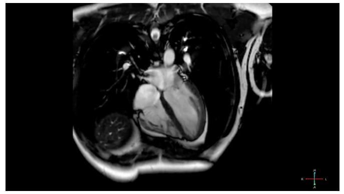
Diffusion is rapid only across small distances



Importance of the physics of fluids

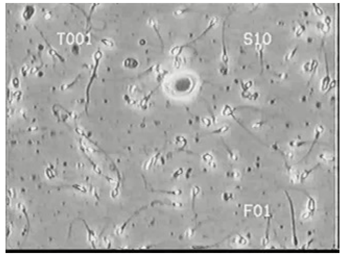
I. Hemodynamics

E.g.: What are the characteristics of blood flow in the circulatory system?
(similarly: flow of gases in respiratory system)



II. Motion in viscous fluids

E.g.: What is the force exerted by a single spermatocyte during its motion?



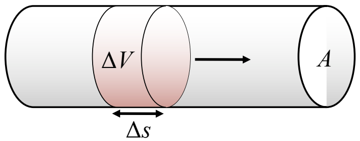
N.B.: gases - in contrast to fluids - are compressible. However, in the physiologically relevant range of pressure differences (~100 Pa), the changes in their volume/density are negligible.

Flow of fluids and gases
Physical properties of blood

- Basic principles
- Types of fluids
- Types of fluid flow
- Laws of fluid flow
- Blood as a fluid; determinants of blood viscosity

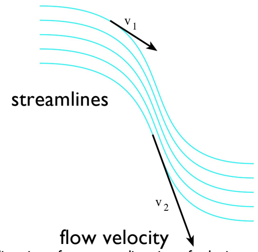
Basic principles I.

Volumetric flow rate (I_V):

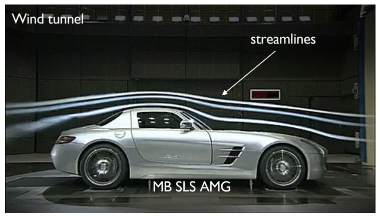


$$I_V = \frac{\Delta V}{\Delta t} = A\frac{\Delta s}{\Delta t} = A\overline{v}$$

Average velocity: $\overline{v} = \frac{I_V}{A}$

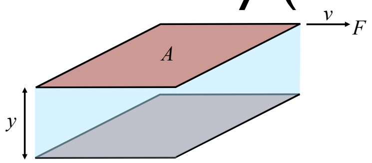


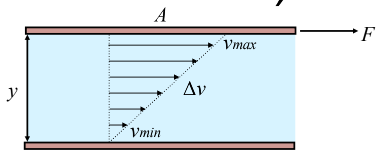
flow velocity
(direction of tangent = direction of velocity
density of streamlines = magnitude of velocity)



Flow can be made visible

Basic principles II.
Viscosity (internal friction)





F = shear force
A = area of fluid layer
η = viscosity
v = flow velocity
y = distance between fluid layers
F/A = shear stress (τ)
$\Delta v/\Delta y$ = velocity gradient (D)

$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y}$$ (Newton's friction law)

$$\eta = \frac{\tau}{D}$$

Units of viscosity: $1Pas = 1\frac{Ns}{m^2} = 10P(poise)$

Viscosity of distilled water (25 ˚C): 1 mPas (1 centipoise)

N.B.: in gases: $\eta \sim T$
Momentum exchange between particles acts against the gliding of layers past one another.

fluids: $\eta \sim e^{E/k_B T}$
Viscosity decreases with the increase in the relative concentration of vacancies.

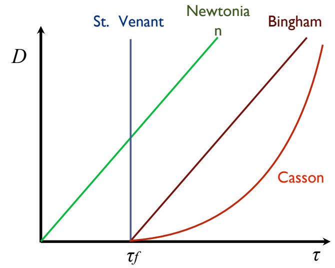
Types of fluids

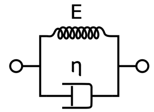
1. *Ideal*
frictionless, incompressible
ρ = constant, η = 0

2. *Non-ideal (real)*
a. Newtonian (viscous)
η independent of shear stress

b. Non-newtonian (anomalous)
η changes with shear stress

Relationship between velocity gradient and shear stress in ***real*** fluids





Model of viscoelastic body - Kelvin-body: spring and dashpot coupled in parallel

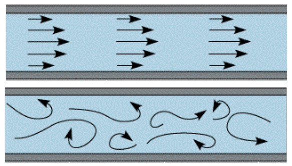
- τ_f = flow threshold
- Viscoelastic materials: combination of viscous and elastic properties (e.g., solution of polymers, macromolecules)
- Stress-relaxation: decay of stress in rapidly deformed viscoelastic body.
- Blood is a non-newtonian fluid; it displays viscoelastic properties.

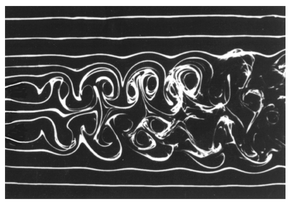
Types of fluid flow

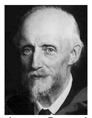
1. **Stationary**
   Volumetric flow rate stays constant (parameters characterizing flow remain unchaned).
2. **Laminar**
   Fluid layers do not mix.
3. **Turbulent**
   Fluid layers mix.

laminar flow

turbulent flow







Osborne Reynolds
(1842-1912)

Reynolds number (Re):

$$\mathrm{Re} = \frac{vr\rho}{\eta}$$

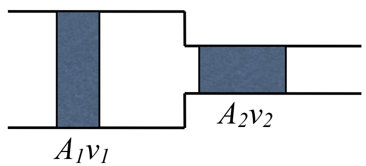
v = flow rate (m/s)
r = tube radius (m)
ρ = density of fluid (kg/m³)
η = viscosity (Ns/m²)

laminar | *turbulent*
Re ≈ 2000

"When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first." (Werner Heisenberg)

Laws of flow in ideal fluids I.

Continuity equation:
volumetric flow rate is constant



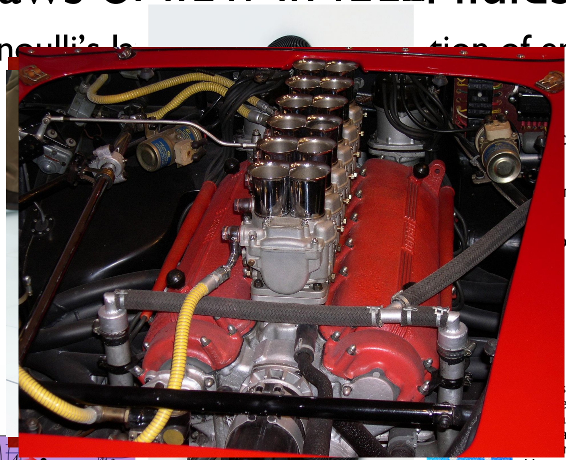
$A_1v_1 = A_2v_2$ = constant

A = cross-sectional area
v = flow velocity

Laws of flow in ideal fluids II.

Berno... ...tion of energy

Daniel Bernoulli (1700-1782)



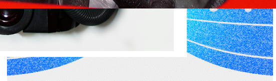
...pressure

...ic pressure

...static pressure

Giovanni Battista Venturi (1746-1822)

Static pressure drops at the tube narrowing



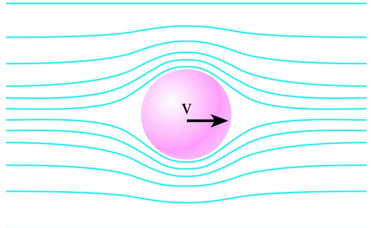
Venturi tube

...ic pulse (*pulsus* ...ens) in aorta insufficiency
...tor (vacuum pump)
...tor (Bunsen-burner)
...rs (volumetric flow rate)
- Vacuum cleaner
- Atomizer, spray, carburetor
- Leadpipe and barrel of wind instruments
- Diffuser

Laws of flow in viscous fluids I.

Stokes' law

Georg Gabriel Stokes (1819-1903)

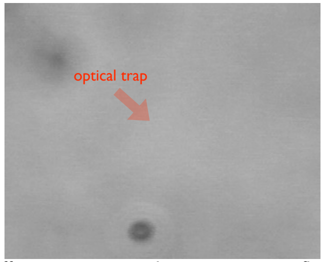


$$F = \gamma v = 6 r \pi \eta v$$

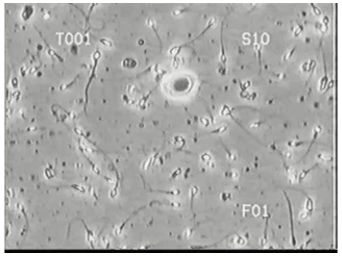
F = force
γ = drag coefficient (shape factor)
v = flow rate
r = radius of sphere
η = viscosity

Stokes force

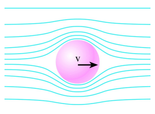
Hydrodynamic drag force (Stokes force): $F = \gamma v = 6 r \pi \eta v$

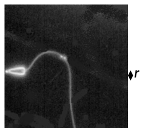


Affects stationary objects in moving fluid (microbead captured in optical trap)



Affects objects moving in stationary fluid (moving spermatocytes)





r = 1.6 μm = 1.6 x 10^{-6} m
v = 50 μm/s = 5 x 10^{-5} m/s
η = 10^{-3} Pas

How much force is exerted by a single spermatotyce during its movement?

$$\gamma = 6 r \pi \eta = 6 \cdot 1.6 \times 10^{-6} \cdot \pi \cdot 10^{-3} = 3 \times 10^{-8} Ns/m$$

$$F = \gamma v = 3 \times 10^{-8} Ns/m \cdot 5 \times 10^{-5} m/s = 1.5 \times 10^{-12} N = \boxed{1.5\,pN}$$

Laws of flow in viscous fluids II.

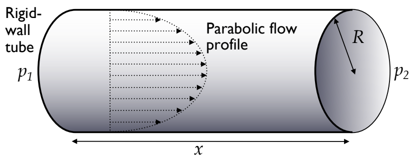
Hagen-Poiseuille's law

G.H.L. Hagen (1797-1884)

J.-L.-M. Poiseuille (1799-1869)

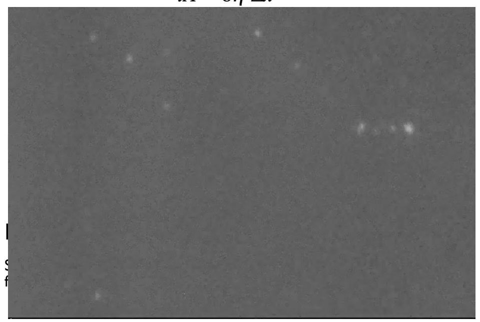
Thermodynamic current	Relevant intensive variable (its difference maintains current)	Current density	Physical law
Volumetric flow	Pressure (p)	$J_V = -\frac{R^2}{8\eta}\frac{\Delta p}{\Delta x}$	Hagen-Poiseuille

$$J_V = \frac{V}{tA} = \frac{R^2}{8\eta}\frac{\Delta p}{\Delta x}$$



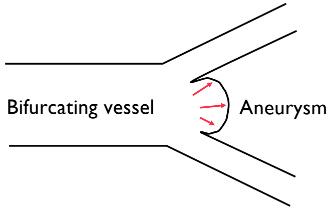
V = volume
t = time
R = tube radius
η = viscosity
p = pressure
x = tube length

$V/t = I_V$ = volumetric vlow rate
$\Delta p/\Delta x$ = pressure gradient, maintained by p_2-p_1 *(negative!)*
A = cross-sectional area of tube
I_V = volumetric flow rate



Medical significance of fluid flow

Bernoulli's law:



Formation of aneurysm (pathological expansion of blood vessel):

- Expansion of vessel: diameter increases
- Flow rate decreases, accoring to continuity equation
- Static pressure increases due to Bernulli's law
- Aneurysm pregrediates - positive fedback mechanism leading to catastrophy

Hagen-Poiseuille's law:

$$\frac{V}{t} = \frac{R^4 \pi}{8\eta} \frac{\Delta p}{\Delta x}$$

Flow intensity, hence the delivered oxygen quantity, may be **drastically reduced** in certain pathological conditions:

- constriction of blood vessels (e.g., diabetes, Bürger's disease)
- change in blood viscosity (e.g., fever, anaemia)
- Reduction of vessel diameter by half leads to a reduction of volumetric flow by 1/16!

Blood as a fluid

55-60% of body mass is water 42 kg (70 kg body mass)		
2/3 intracellular 28 kg	1/3 extracellular 14 kg	
	1/3 plasma 4-5 kg	2/3 intersticium 9-10 kg

Blood:

Average volume: 5 l
Average viscosity: 5 mPas
Average density: 1.05 g/cm^3
Composition: 40-45 % corpuscular, 55-60 % plasma

Determinants of blood viscosity

1. Hematocrit (htc, ϕ):

$$htc = \frac{V_{cells}}{V_{total}}$$

Normal range: 0.4-0.5.

Viscosity of blood as suspension
(in the physiologically relevant htc range):

$$\lg \eta_s = A + B\phi$$

η_s=suspension viscosity
A, B=empirical constants

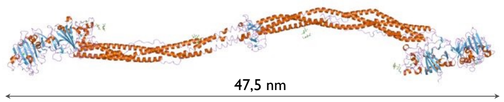
Determinants of blood viscosity

2. Plasma viscosity

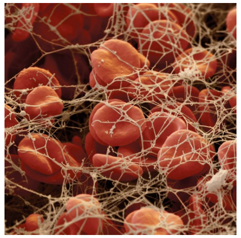
- Depends on plasma proteins.
- In *paraproteinaemias* (e.g. myeloma multiplex or plasmocytoma) the concentration of immunoglobulins is high, leading to increased viscosity.

Plasma protein	Normal concentration	% ratio	Function
Albumin	35-50 g/l	55%	maintenance of colloind osmotic pressure, transport
Globulins	20-25 g/l	38%	Part of the immune system
Fibrinogen	2-4.5 g/l	7%	Blood coagulation

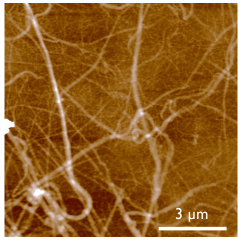
Fibrinogen, fibrin



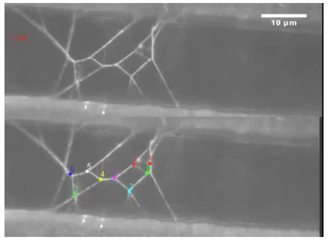
Fibrinogen:
MW = 340.000 Da
In plasma: 2-4 g/l ≈ **10 μM**
Average nearest-neighbor distance ≈ **55 nm**



Red blood cells in fibrin meshwork



Fibrin polymerized in vitro (AFM)

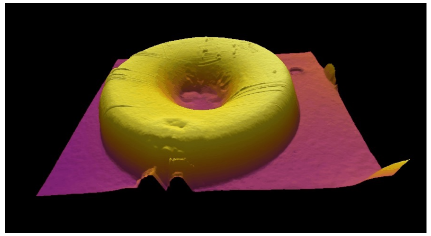


Extensibility of fluorescently labeled fibrin fibrils

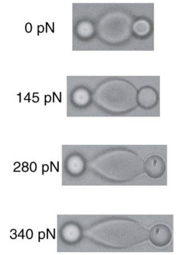
Determinants of blood viscosity

3. Plasticity of red blood cells

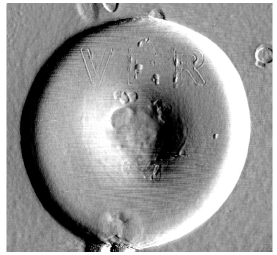
- 65% suspension of blood-cell-size particles is rock hard.
- By contrast, a 95% blood suspension if fluid, with viscosity of ~20 mPas!
- Deformation of red blood cells: droplet, parachute, arrowhead shapes.



Disc-shaped cell with 7-11 μm diameter



Deformation of a RBC with optical tweezers

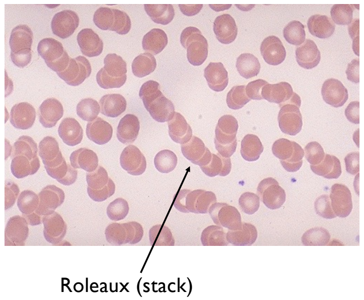


Fixed RBC maintaining impression (AFM)

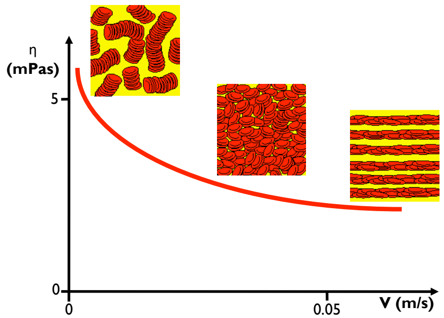
Determinants of blood viscosity

4. Aggregation of red blood cells

- Stack or roleaux formation.
- More pronounced at low flow rates

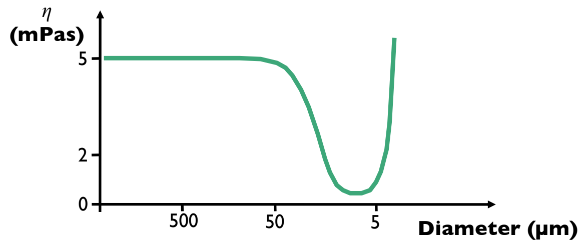


5. Flow rate, velocity gradient



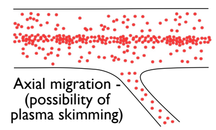
Determinants of blood viscosity

6. Blood vessel diameter



N.B.:

- With a decrease of vessel diameter, the anomalous (non-newtonian) behavior of blood becomes more pronounced.
- *Axial migration*: the red blood cells line up in the axis of the vessel (Bernoulli's law). In the axis the velocity gradient decreases, and near the vessel wall it increases. Increase in velocity gradient decreases apparent viscosity (Fåhraeus-Lindquist effect).



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