Transport processes-1
fluid flow

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Transport = translocation of materials

biological significance: intracellular, intercellular, and transmembrane flow of matter (breathing, blood circulation, transport across membranes, metabolism)

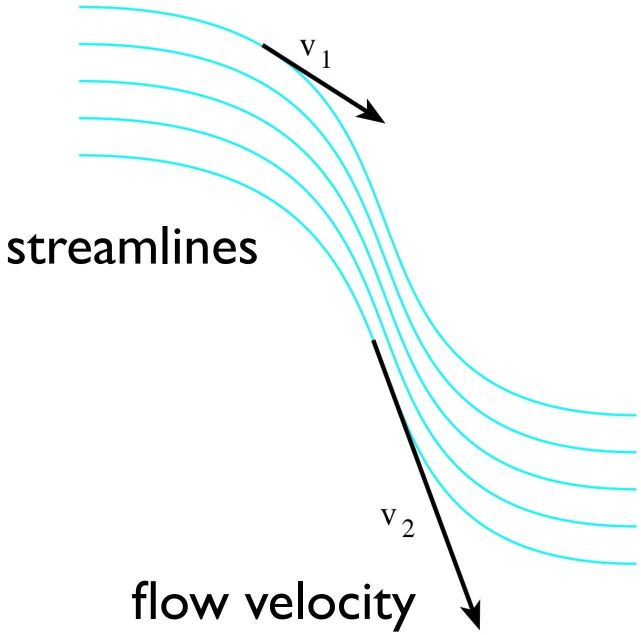
Distinct transport mechanisms:

– Transport is due to a collective, directional motion of particles, such as in a **fluid flow**

– Trasport is due to independent random motion of particles, such as **diffusion**

– Transporting materials through lipid bilayers – **membrane transport**

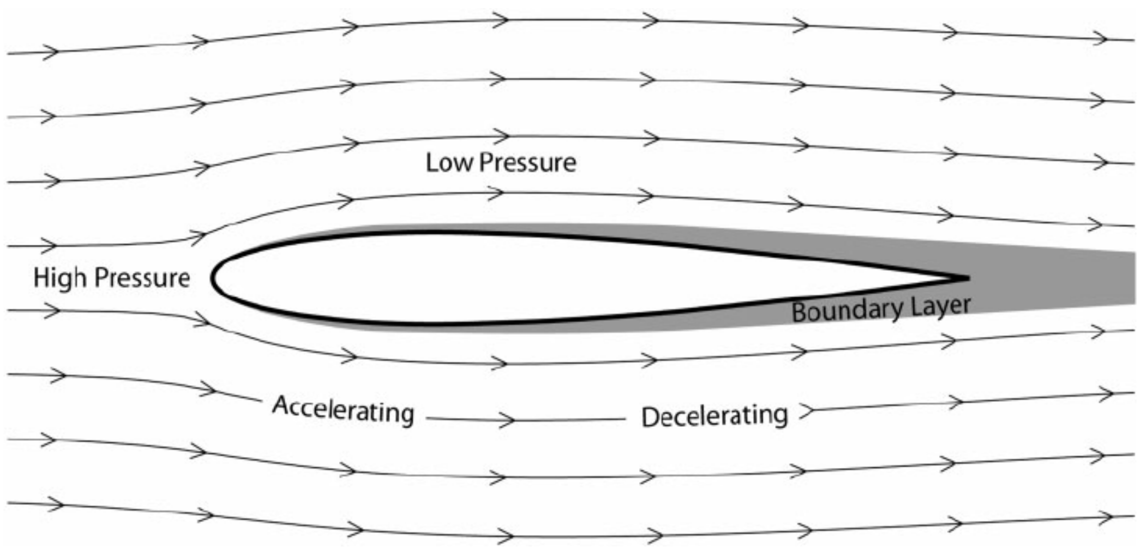
Macroscopically observable fluid flow



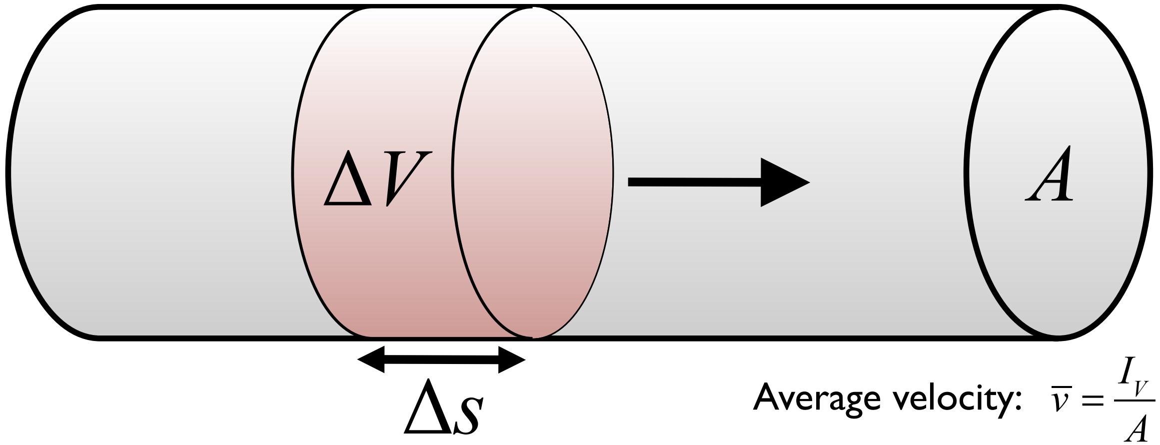
(direction of tangent = direction of velocity
density of streamlines = magnitude of velocity)

Small drag in streamlined position

Large drag in unstreamlined position



Flow in rigid-wall tubes



Average velocity: $\bar{v} = \frac{I_V}{A}$

Volumetric flow rate (I_V):

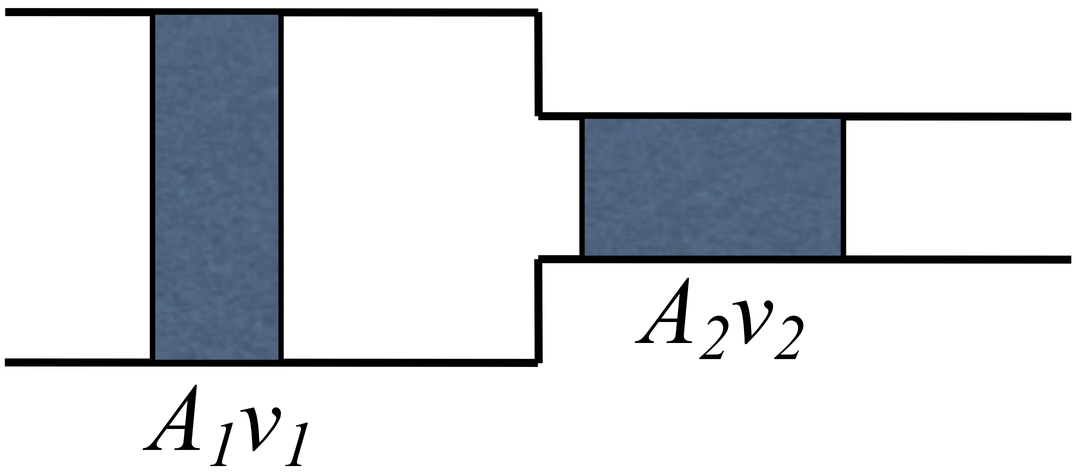
$$I_V = \frac{\Delta V}{\Delta t} = A\frac{\Delta s}{\Delta t} = A\overline{v}$$

Stacionary flow

parameters of flow are **constant** in time

Continuity principle: in a stacionary flow the volumetric flow rate is constant

e.g. law of mass balance for incompressible fluids (gases?)

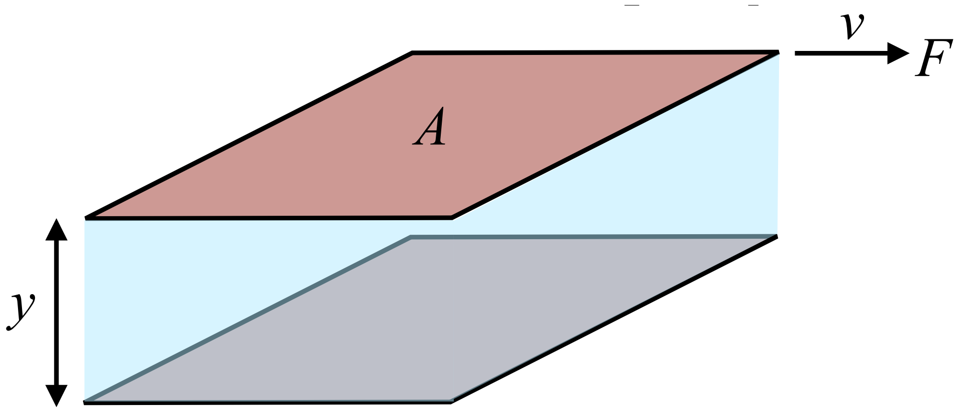


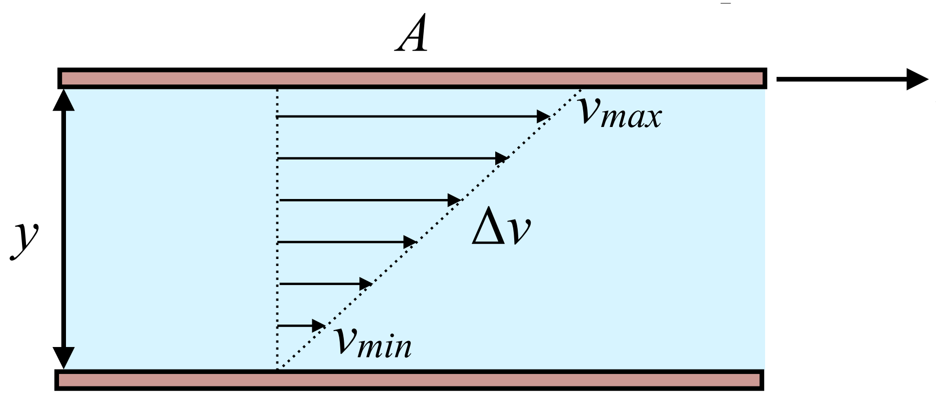
$$A_1v_1 = A_2v_2 = \text{constant}$$

A = cross-sectional area

v = flow velocity

Viscosity – internal friction





F	= shear force
A	= area of fluid layer
η	= viscosity
v	= flow velocity
y	= distance between fluid layers
F/A	= shear stress (τ)
$\Delta v/\Delta y$	= velocity gradient (D)

$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y}$$ (Newton's friction law)

$$\eta = \frac{\tau}{D}$$

Units of viscosity: $1 Pas = 1 \frac{Ns}{m^2} = 10 P (poise)$

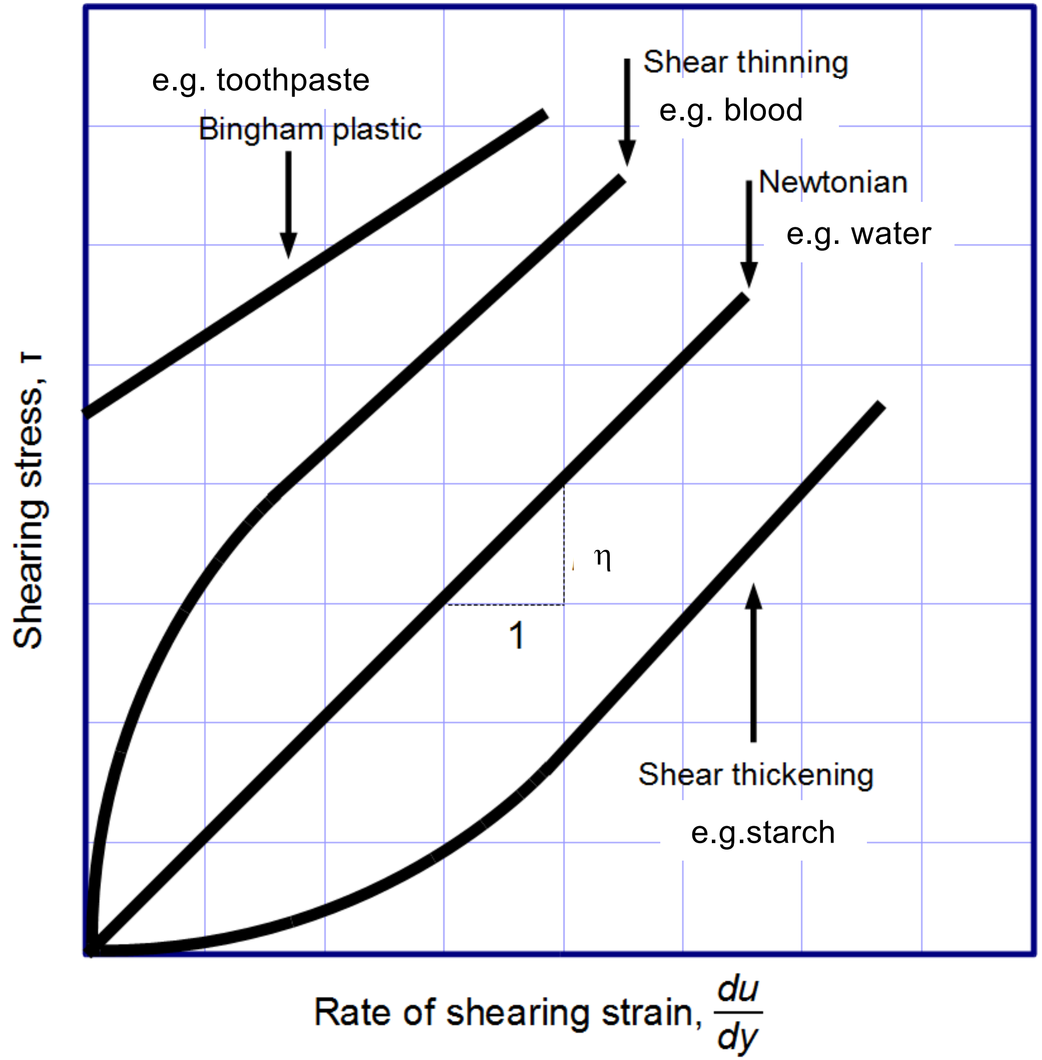
Viscosity of distilled water (25 °C): 1 mPas (1 centipoise)

Types of fluids based on viscosity

1. Ideal
frictionless, incompressible
ρ = constant, η = 0

2. Non-ideal (real)
a. Newtonian (viscous)
η independent of shear stress

b. Non-newtonian (anomalous)
η changes with shear stress



$$\eta = \tau / D$$

Viscosity – internal friction

substance	η (mPa·s) 20 °C
air	(101 kPa) 0.019
water	1
ethanol	1.2
blood (37 °C)	2–8
glycerine	1490
honey	2000–14000

viscosity of gases increases with rising temperature

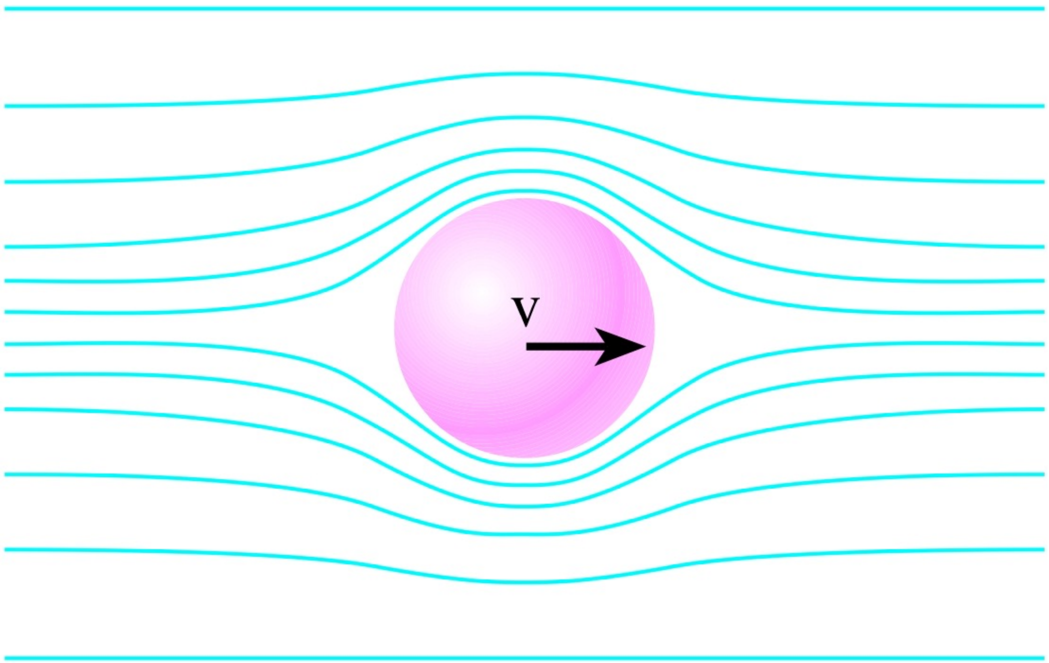
viscosity of fluids decreases with rising temperature

$$\eta \sim e^{\frac{E}{kT}}$$

$$\frac{F}{A} = \eta \frac{\Delta v}{\Delta y}$$

Fluids obeying the above equation are called **newtonian fluids**.

Friction on spherical particles – Stokes law



Georg Gabriel Stokes
(1819-1903)

Frictional force is proportional with the velocity:

F = force
γ = drag coefficient (shape factor)
v = flow rate
r = radius of sphere
η = viscosity

$$F = \gamma v = 6\pi\eta r v$$

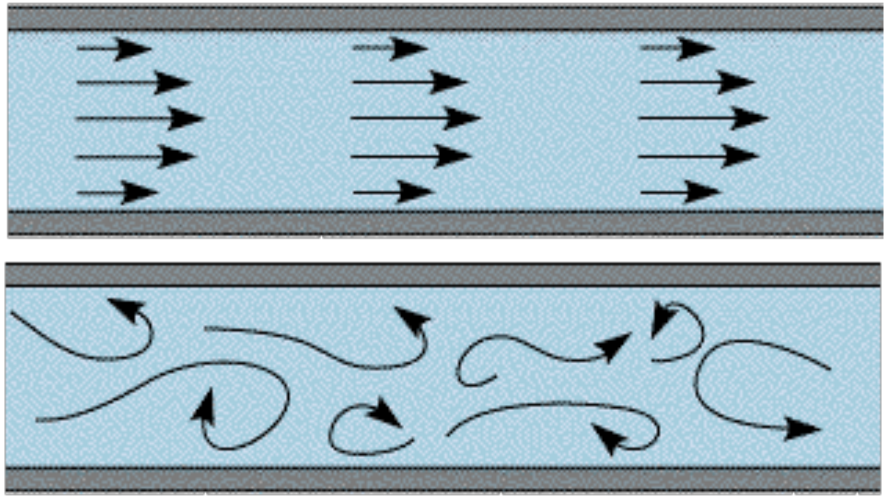
Types of flow:

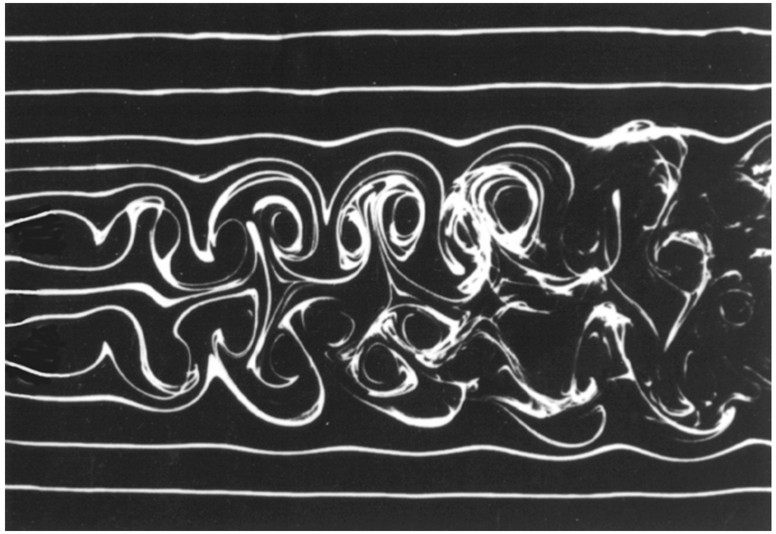
laminar flow – fluid layers do not mix

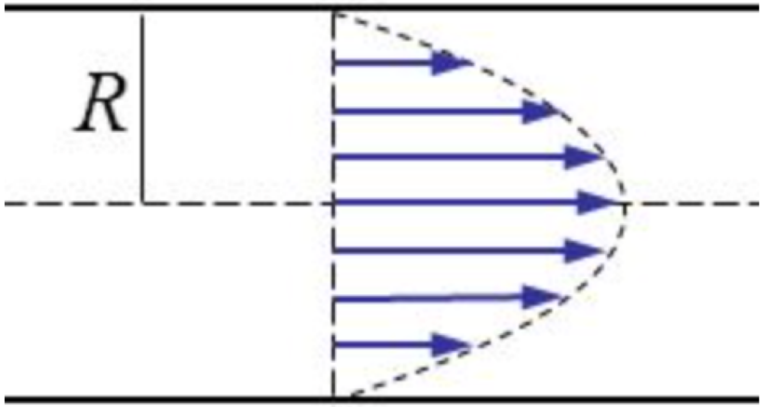
turbulent flow – layers are mixing (chaotic streamlines)

laminar flow

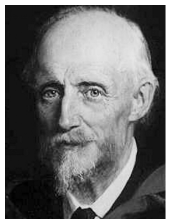
turbulent flow







parabolic velocity profile



Osborne Reynolds
(1842-1912)

Reynolds number (Re):

$$\mathrm{Re} = \frac{v r \rho}{\eta}$$

v = flow rate (m/s)
r = tube radius (m)
ρ = density of fluid (kg/m^3)
η = viscosity (Ns/m^2)

$\mathrm{Re} \approx 2000$

Re > 1000 – turbulent flow appears

Bernoulli's law

conservation of energy in ideal fluids

Daniel Bernoulli
(1700-1782)

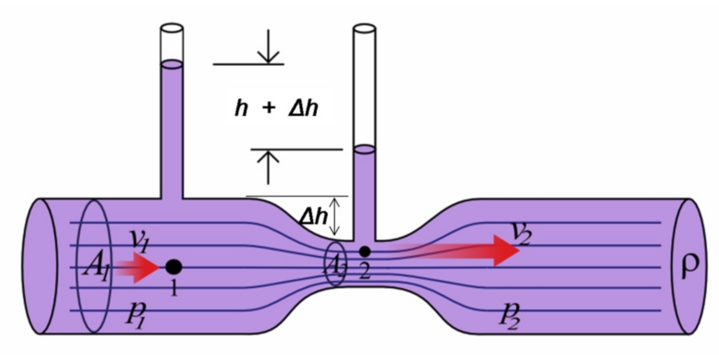
$$p + \frac{1}{2}\rho v^2 + \rho g h = \ const$$

p = static pressure

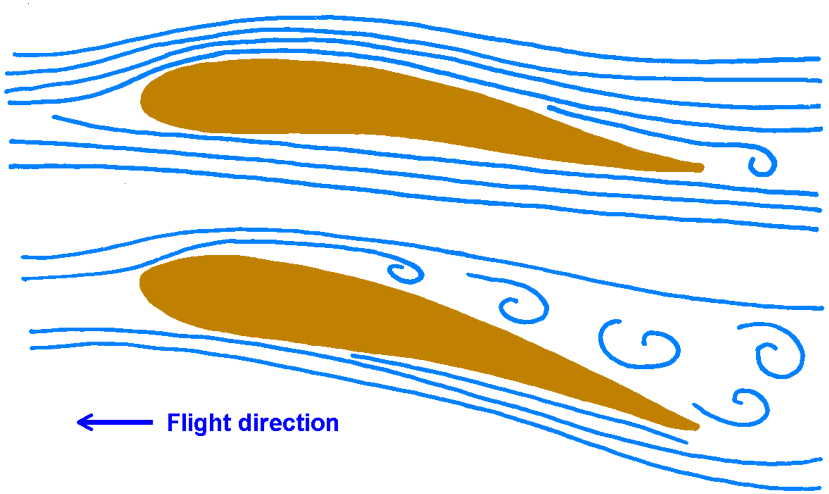
$\frac{1}{2}\rho v^2$ = dynamic pressure

$\rho g h$ = hydrostatic pressure

Giovanni Battista
Venturi
(1746-1822)



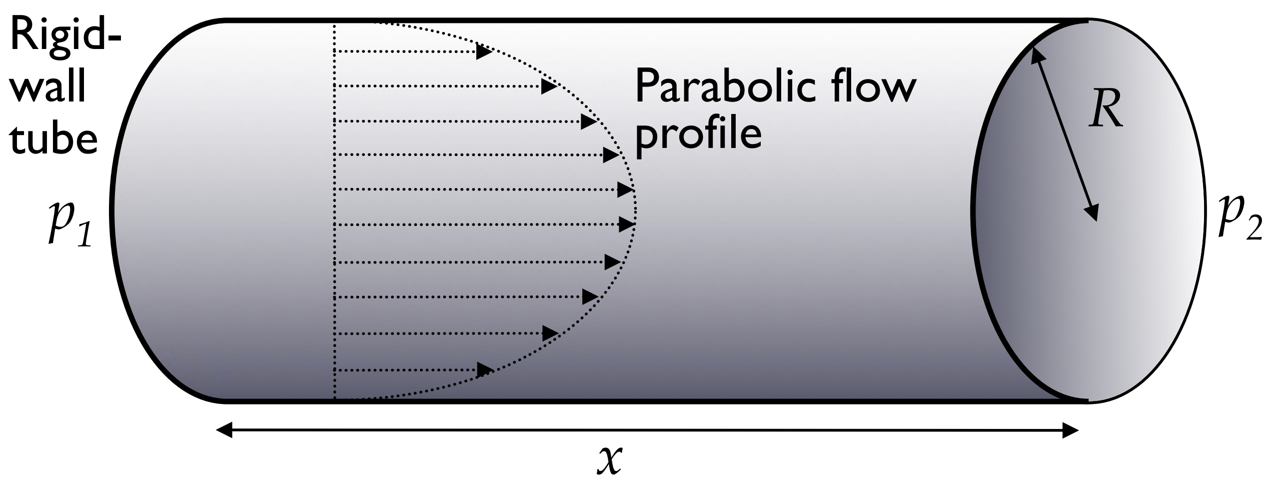
Static pressure drops at the
tube narrowing



Applications: sprayer, atomizer, aspirator (vacuum pump), inspirator (Bunsen burner), diffuser, jet engine, wing profile, vacuum cleaner, biphasic pulse in aorta insufficiency

Hagen-Poiseuille's law

stacionary laminar flow of newtonian fluids and gases in rigid-wall tubes



G.H.L. Hagen
(1797-1884)

J.-L.-M. Poiseuille
(1799-1869)

V = volume
t = time
R = tube radius
η = viscosity
p = pressure
x = tube length

$V/t = I_V$ = volumetric vlow rate
$\Delta p/\Delta x$ = pressure gradient, maintained by p_2-p_1 *(negative!)*
A = cross-sectional area of tube
I_V = volumetric flow rate

$$I_V = \frac{V}{t} = -\frac{R^4\pi}{8\eta}\frac{\Delta p}{\Delta x}$$

$$I_V = -\frac{R^4\pi}{8\eta\Delta x}\Delta p \Rightarrow -\Delta p = R_{tube} \cdot I_V \Rightarrow U = R \cdot I$$

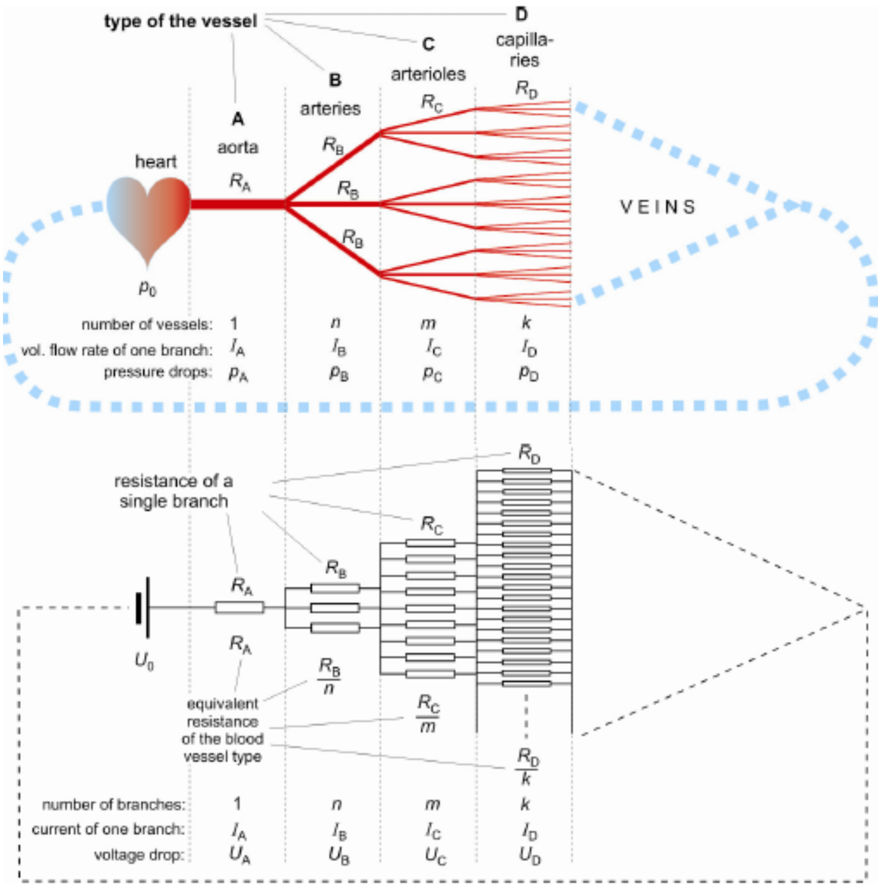
$1/R_{tube}$

Ohm's law!

Electric model of the arterial vascular system

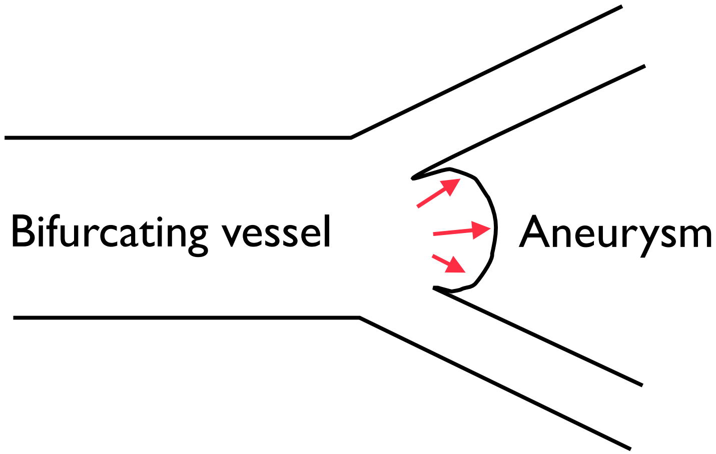
volume flow rate $(I_V) = \dfrac{\text{pressure drop } (\Delta p)}{\text{resistance to flow } (R_{\text{flow}})}$	electric current $(I) = \dfrac{\text{voltage } (U)}{\text{resistance } (R)}$

static pressure drop (Δp)	electric voltage, potential drop (U)
resistance to flow (R_{flow})	resistance (R)
volume flow rate (I_V)	electric current (I)



Medical significance of fluid flow laws

Bernoulli's law:



Formation of aneurysm (pathological expansion of blood vessel):

- Expansion of vessel: diameter increases
- Flow rate decreases, accoring to continuity equation
- Static pressure increases due to Bernulli's law
- Aneurysm pregrediates - positive fedback mechanism leading to catastrophy

Hagen-Poiseuille's law:

$$\frac{V}{t} = \frac{R^{\boxed{4}}\pi}{8\eta}\frac{\Delta p}{\Delta x}$$

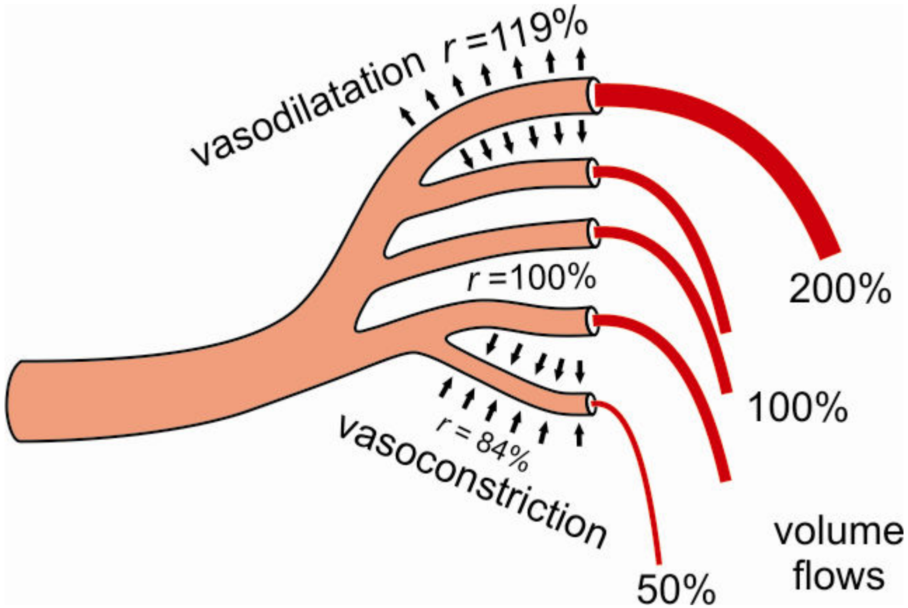
Flow intensity, hence the delivered oxygen quantity, may be **drastically reduced** in certain pathological conditions:

- constriction of blood vessels (e.g., diabetes, Bürger's disease)
- change in blood viscosity (e.g., fever, anaemia)
- Reduction of vessel diameter by half leads to a reduction of volumetric flow by 1/16!

Medical significance of fluid flow laws

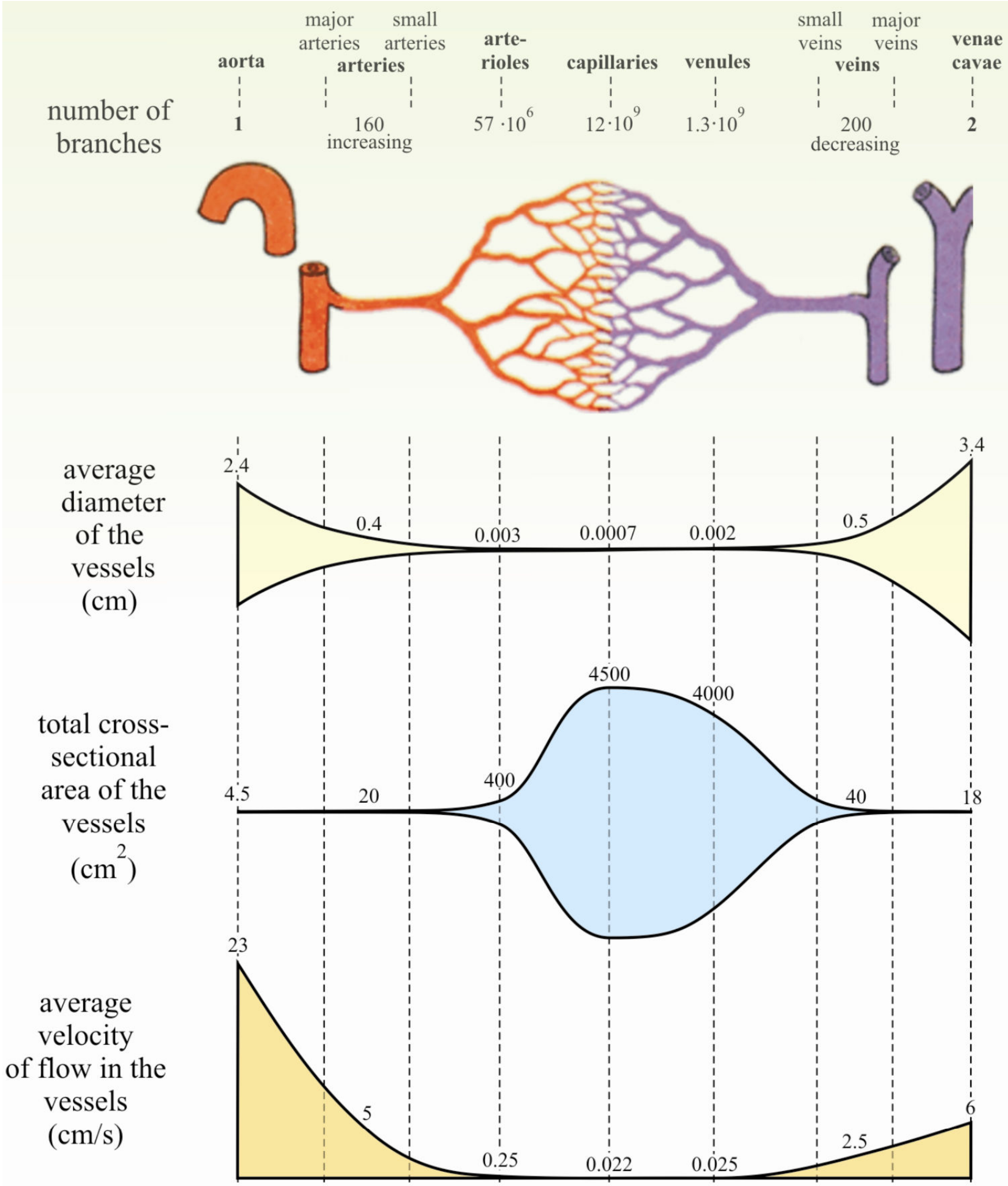
regulation of the intensity of blood flow
(Hagen-Poiseuille's law)

$$I_V = \frac{V}{t} = \frac{R^4 \pi}{8\eta} \frac{\Delta p}{\Delta x}$$

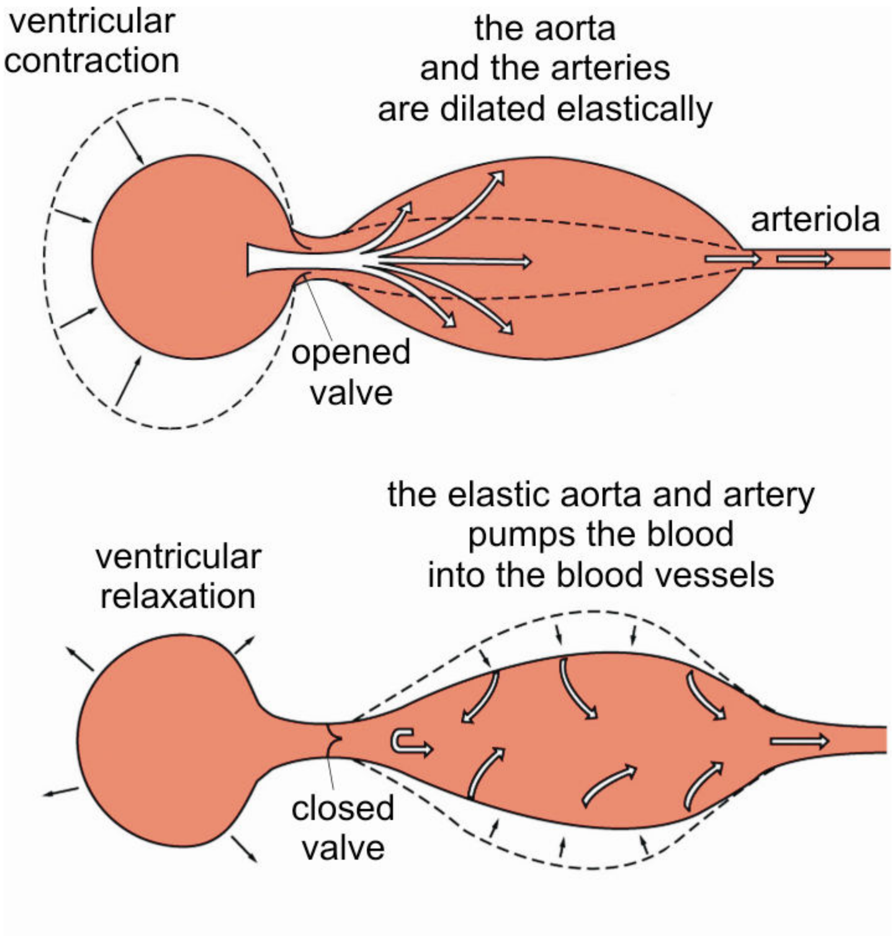


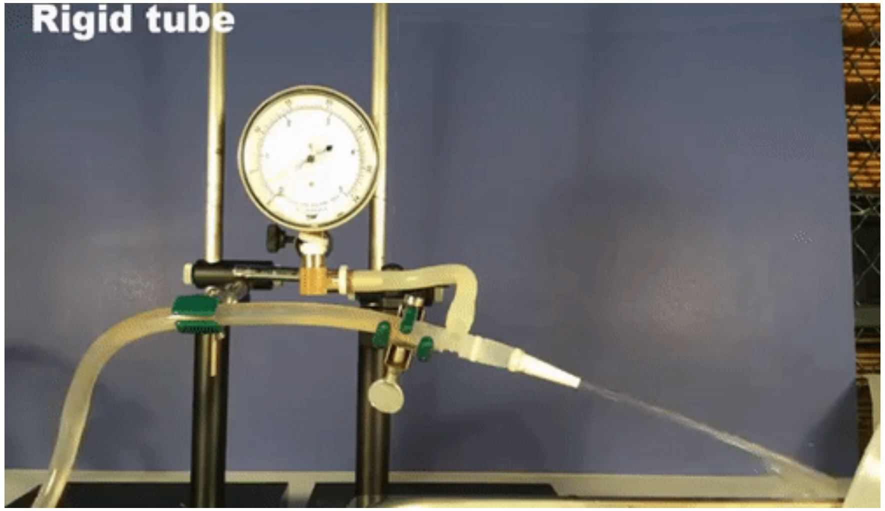
Physiological regulation of the blood flow depending on the needs of tissues/organs

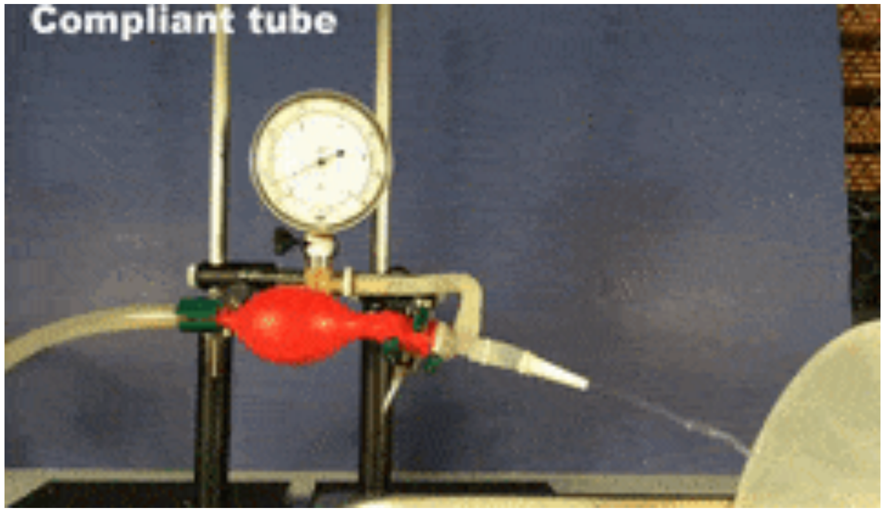
Flow parameters in the circulation



Flow conditions in the arterial system: pulsatile flow, elastic arteriall wall







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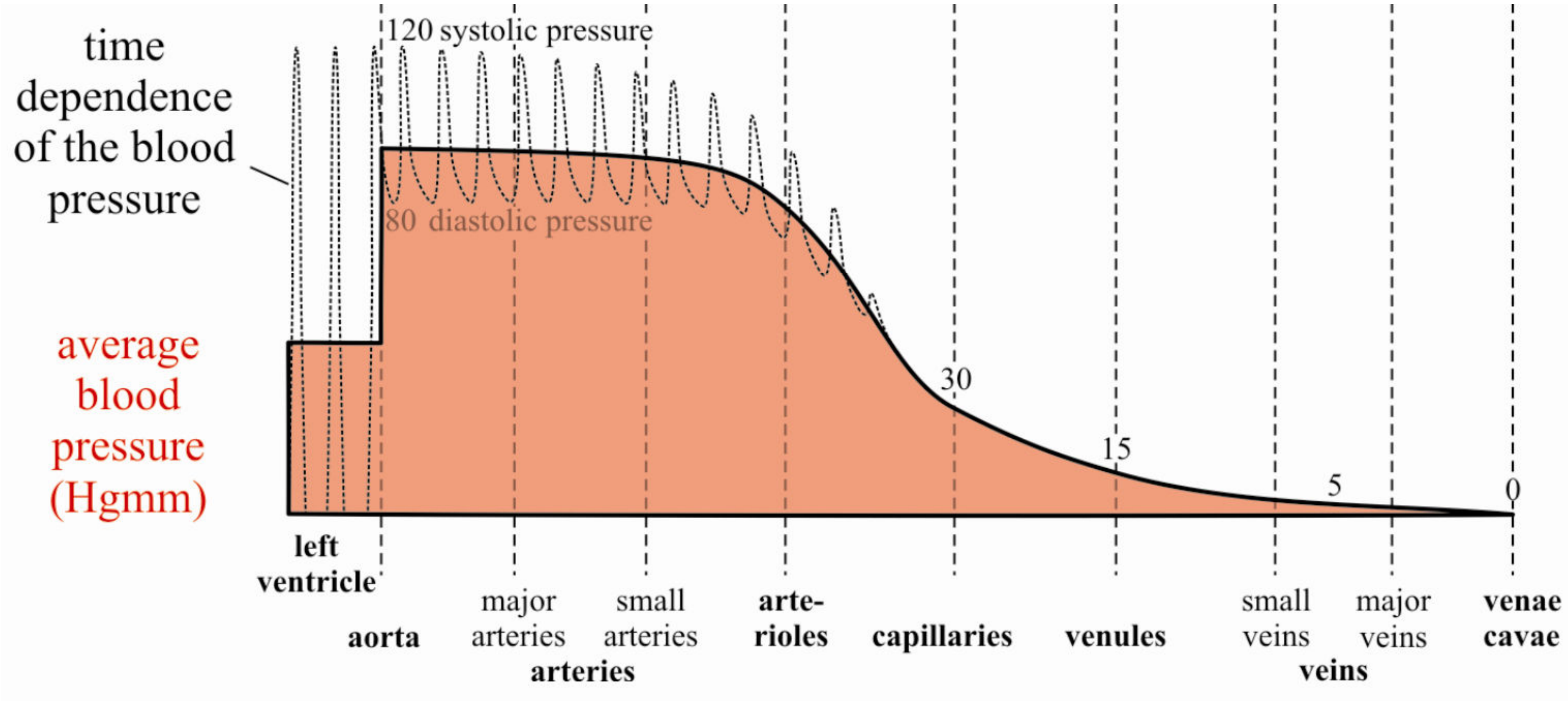


Viscosity of blood

55-60% of body mass is water 42 kg (70 kg body mass)		
2/3 intracellular 28 kg	1/3 extracellular 14 kg	
	1/3 plasma 4-5 kg	2/3 intersticium 9-10 kg

Blood:

Average volume: 5 l
Average viscosity: 5 mPas
Average density: 1.05 g/cm^3
Composition: 40-45 % corpuscular, 55-60 % plasma

Viscosity of blood

1. Hematocrit (htc, ϕ):

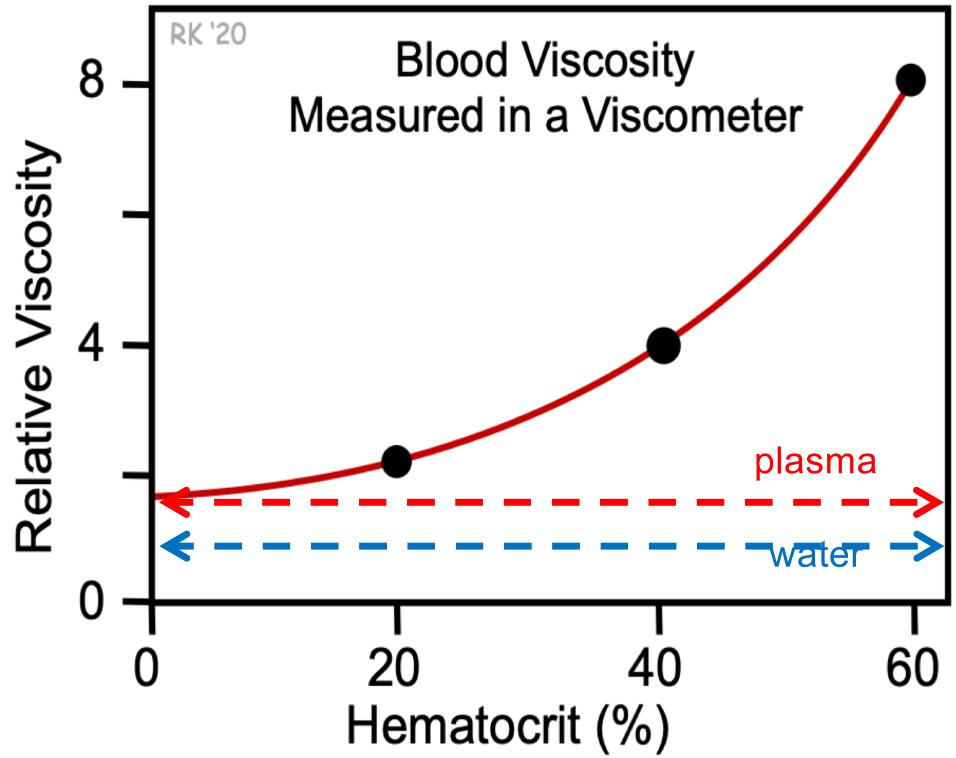
$$htc = \frac{V_{cells}}{V_{total}}$$

Normal range: 0.4-0.5.

Viscosity of blood as suspension
(in the physiologically relevant htc range):

$$\lg \eta_s = A + B\phi$$

η_s=suspension viscosity
A, B=empirical constants



Viscosity of blood

2. *Plasma viscosity*

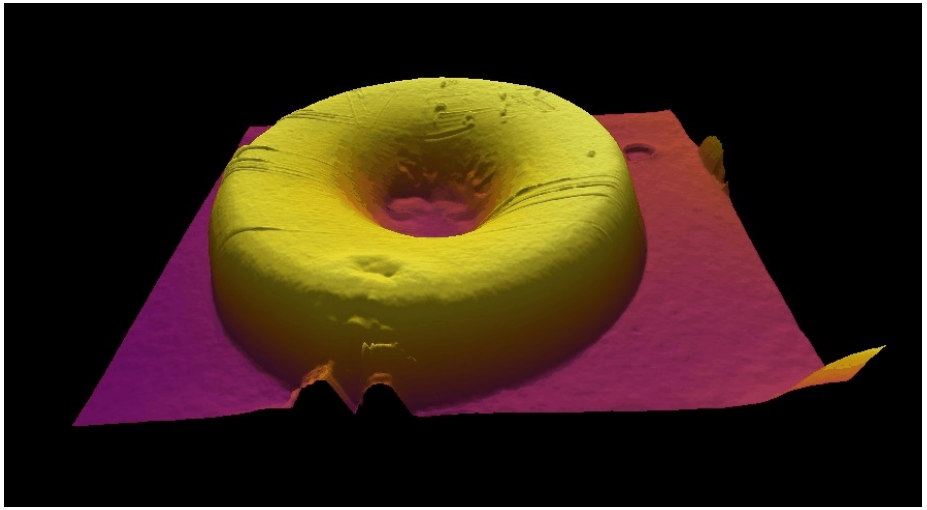
- Depends on plasma proteins.
- In *paraproteinaemias* (e.g. myeloma multiplex or plasmocytoma) the concentration of immunoglobulins is high, leading to increased viscosity.

Plasma protein	Normal concentration	% ratio	Function
Albumin	35-50 g/l	55%	maintenance of colloind osmotic pressure, transport
Globulins	20-25 g/l	38%	Part of the immune system
Fibrinogen	2-4.5 g/l	7%	Blood coagulation

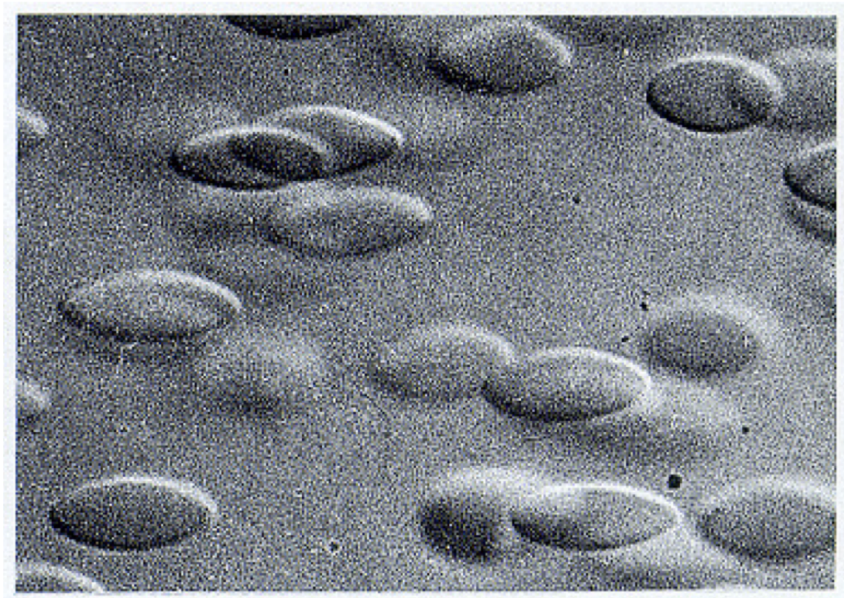
Viscosity of blood

3. Plasticity of red blood cells

- 65% suspension of blood-cell-size particles is rock hard.
- By contrast, a 95% blood suspension if fluid, with viscosity of ~20 mPas!
- Deformation of red blood cells: droplet, parachute, arrowhead shapes.



Disk-shaped cells with 7-11 um diameter



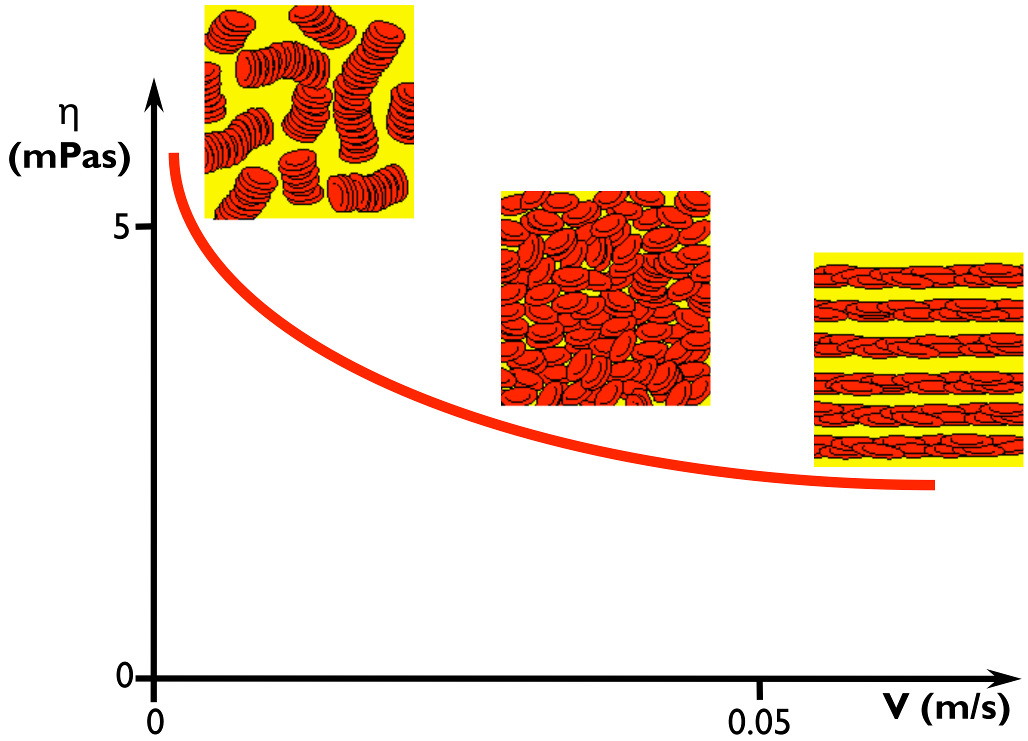
Viscosity of blood

4. Aggregation of red blood cells

- Stack or roleaux formation.
- More pronounced at low flow rates

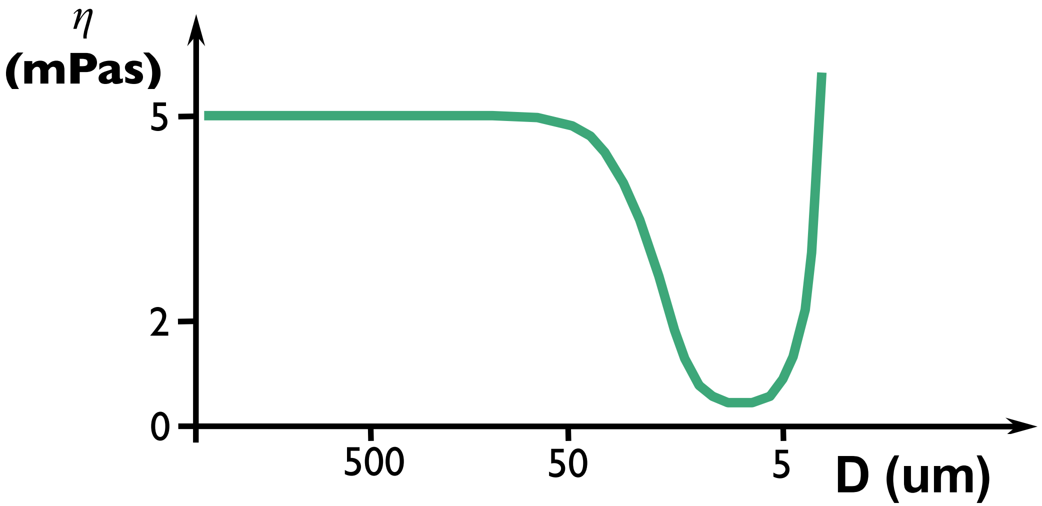
Roleaux (stack)

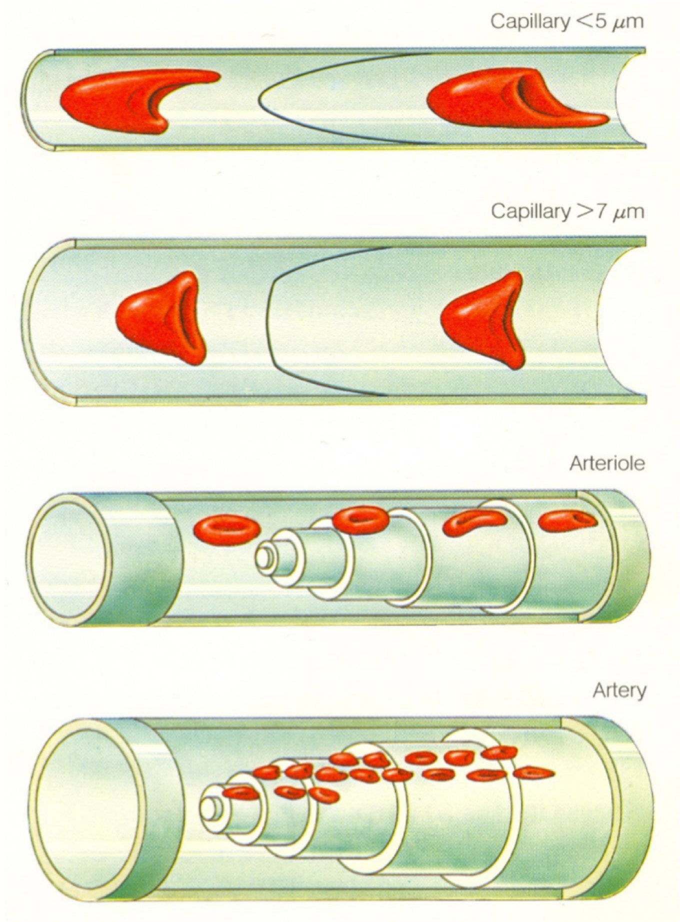
5. Flow rate, velocity gradient



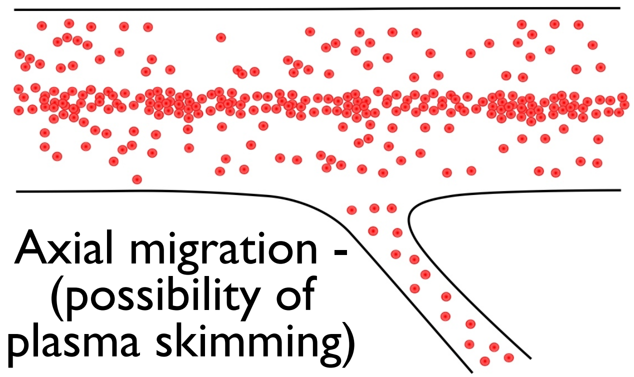
Viscosity of blood

6. Blood vessel diameter

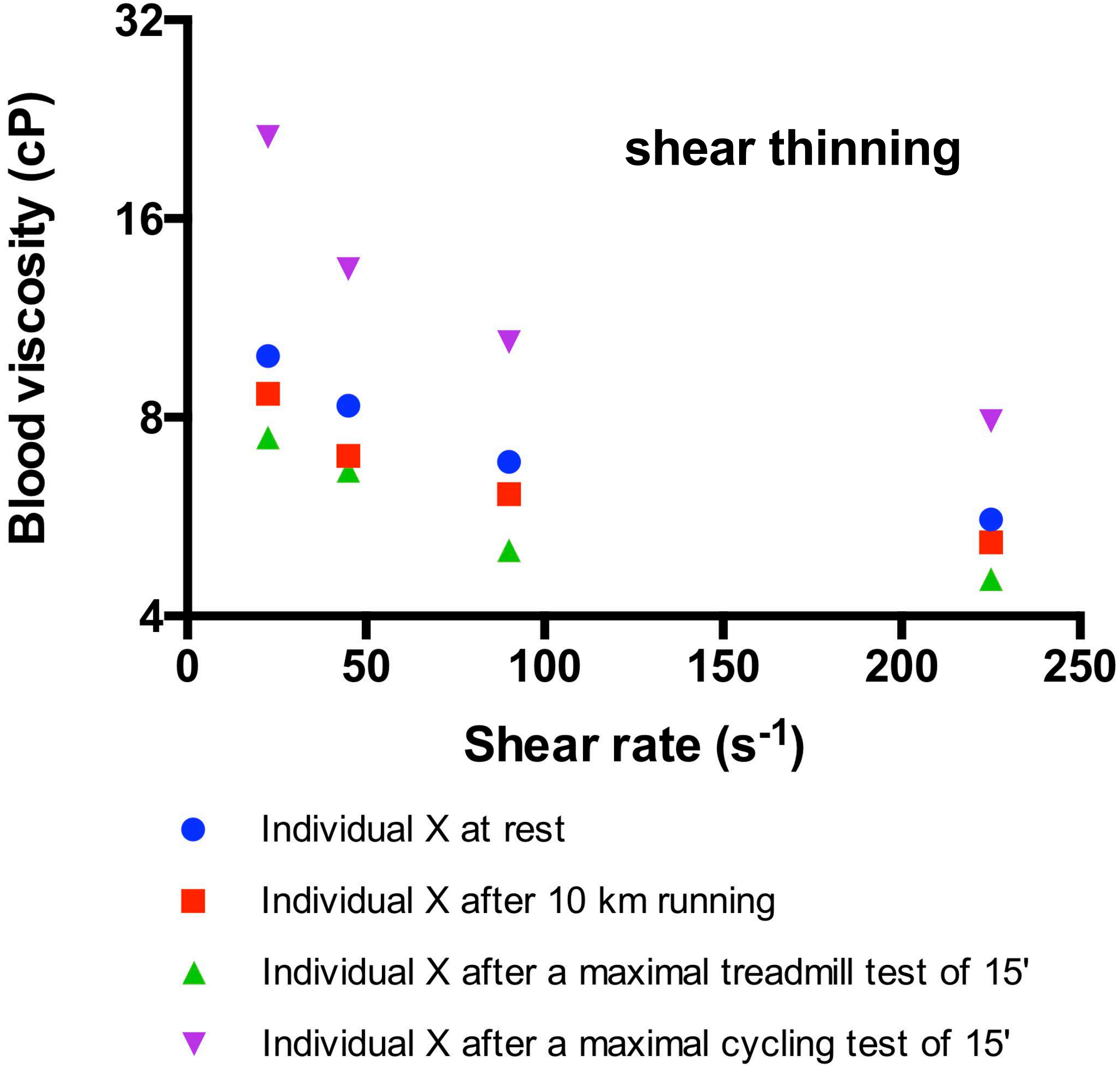


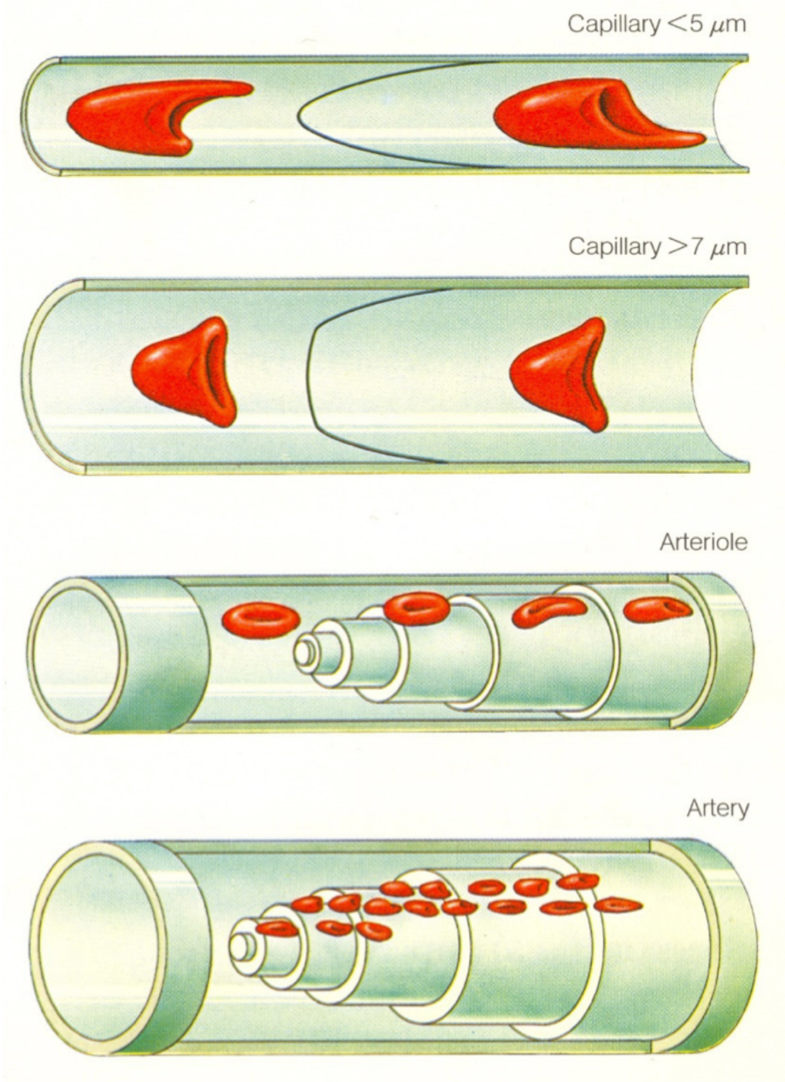


- With a decrease of vessel diameter, the anomalous (non-newtonian) behavior of blood becomes more pronounced.
- *Axial migration*: the red blood cells line up in the axis of the vessel (Bernoulli's law). In the axis the velocity gradient decreases, and near the vessel wall it increases. Increase in velocity gradient decreases apparent viscosity (Fåhraeus-Lindquist effect).

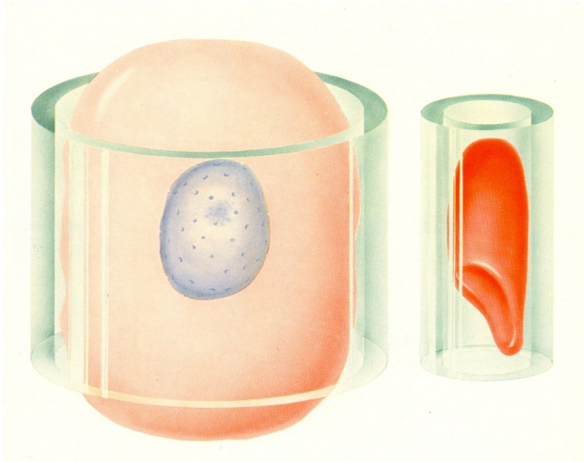


Adaptation of RBC due to deformability



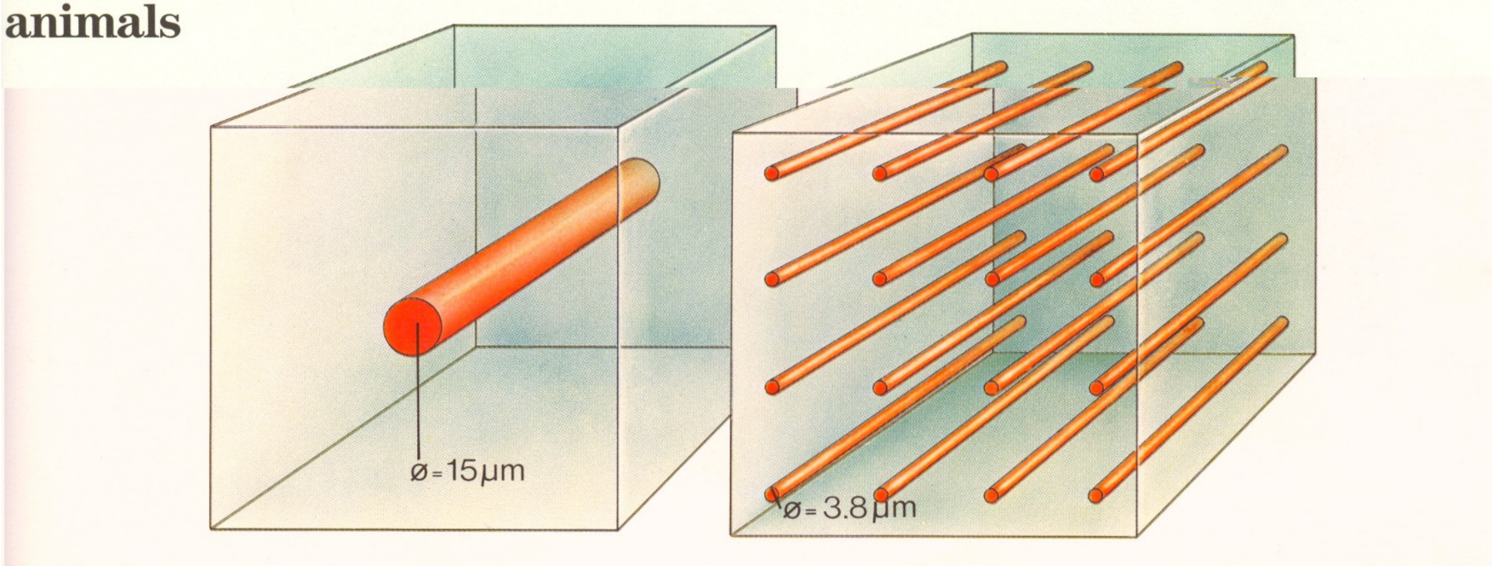


Viscosity of blood



Schematic diagram of capillary formation in cold-blooded animals

Schematic diagram of capillary formation in mammals



Related chapters:

Damjanovich, Fidy, Szöllősi: Medical biophysics

III./1.
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1.5

Practicals: Flow