Electromagnetism III

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Lorenz force

Lorenz force: is the force acting on a charge Q moving with velocity v in a static magnetic field.

$F = B \cdot I \cdot l$

$I = \frac{Q}{t}$ $\quad \frac{l}{t} = v$

$$F = B \cdot \frac{Q}{t} \cdot l = B \cdot Q \cdot v$$

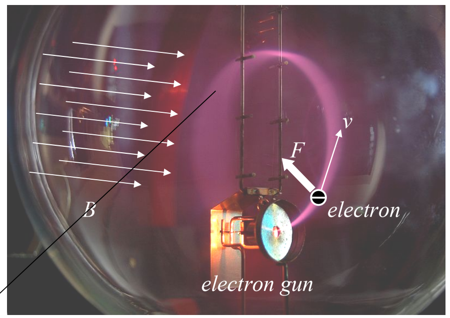
$B \perp v \perp F$

⇓

F will be a centripetal force

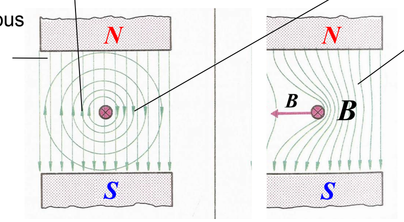
⇓

the charge will move on a circular path



Forces in magnetic fields

The magnetic field of the electric current and the static magnetic field superimpose. In some places they weaken, in others they strengthen each other.



The current-carrying wire will experience a force that is pushing it towards the weaker magnetic field.

induction: $F \sim B$

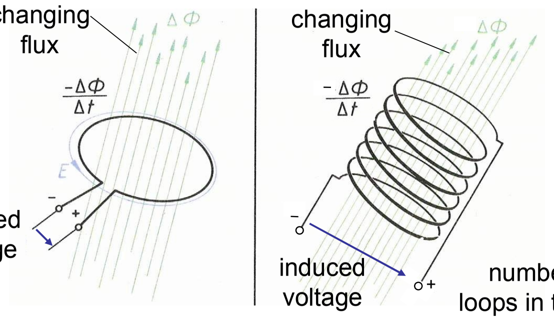
current intensity: $F \sim I$

length of the wire: $F \sim l$

force: $F = B \cdot I \cdot l$

Faraday's law of induction

The electromotive force (voltage) produced around a closed path is proportional to the rate of change of the magnetic flux through the surface bounded by that path.

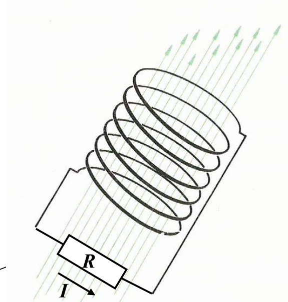


$U_i = -\Delta\Phi/\Delta t$ (flux change; time during which the flux change happened)

$U = -N \cdot \Delta\Phi/\Delta t$ (number of loops in the coil)

Lenz's law

An induced current is always in such a direction as to oppose the motion or change causing it.
(this is what the ”-” sign means in Faraday's law)



flux generated by the induced current

U induced electron motive force

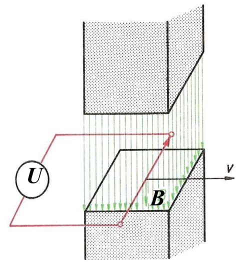
R resistance, $\boldsymbol{I = U / R}$ induced current

Motional electron motive force

A straight conductor with length l is moved in a homogeneous magnetic field with a velocity v perpendicular to the magnetic induction vector.

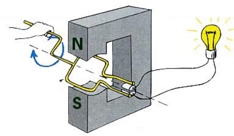
$$Q \cdot E = B \cdot Q \cdot v$$
$$E = B \cdot v$$
$$U = l \cdot E = l \cdot B \cdot v$$

induced voltage: $\boldsymbol{U = B \cdot l \cdot v}$



electric generator:

mechanic energy in



electric energy out

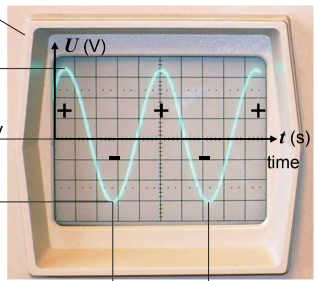
Alternating current (AC)

Parameters describing a sinusoidal alternating voltage:

voltage amplitude (V) angular frequency (rad/s)

voltage (V) $U = U_{peak} \cdot \sin \omega t$ time (s)

oscilloscope screen



$U_{peak-peak}$ U_{peak} T

frequency (Hz=1/s)

$\omega = 2\pi f = 2\pi/T$ period time (s)

$f = 1/T$

voltage amplitude (V) period (s)

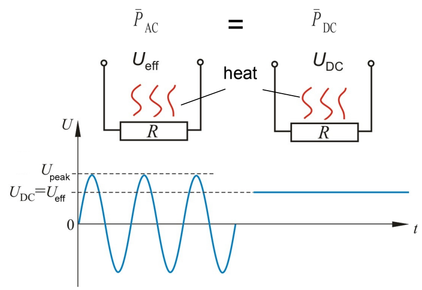
peak-to-peak voltage (V) $U_{peak} = U_{peak-peak}/2$ $f = 50$ Hz

e.g. for the voltage of the mains: $T = 1/f = 1/50\text{Hz} = 0.02\text{s} = 20\text{ms}$

Effective value

The effective value of an AC current (voltage) is equivalent to the DC current (voltage) that on average produces the same amount of heat on a resistor.

For sinusoidal voltage and current:



$$U_{eff} = \frac{U_{peak}}{\sqrt{2}}$$

$U_{eff} = 0{,}707 \cdot U_{peak}$

$U_{peak} = 1{,}41 \cdot U_{eff}$

$$I_{eff} = \frac{I_{peak}}{\sqrt{2}}$$

Effective value of the mains electricity

230V sinusoidal voltage

Effective value is used and not the peak voltage!

$U_{eff} = 230\ \text{V}$

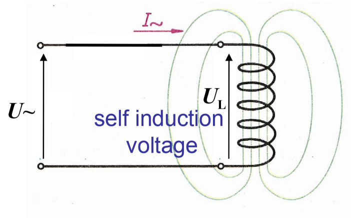
$U_{peak} = 1{,}41 \cdot 230\ \text{V} = 324\ \text{V}$

The advantage of using effective value rather than peak value is that the effective values can be directly used for calculating power.

$\bar{P} = U_{eff} \cdot I_{eff}$ but $\bar{P} \neq U_{cs} \cdot I_{cs}$

Self induction

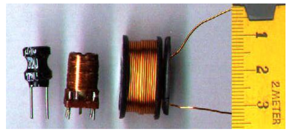
- We connect an AC voltage to a coil.
- A constantly changing magnetic field is generated around the coil.
- The changing magnetic field induces a voltage in the coil that opposes the change of the external voltage.



self induction voltage — $U_L \sim \frac{\Delta I}{\Delta t}$ — current change / time during which the current change happened

$U_L = -L \cdot \frac{\Delta I}{\Delta t}$

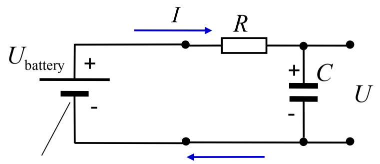
The proportionality factor L is called self inductance



unit of self inductance: henry, H

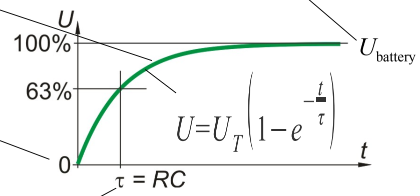
1H is the self inductance of the coil which responds to a change of 1A that happens uniformly during 1s with 1V self induction voltage.

Charging a capacitor through a resistor



DC voltage generator (e.g. battery)

After long time the capacitor will be charged to the voltage of the battery: $U = U_{battery}$ (Equilibrium is reached and no current will flows.)



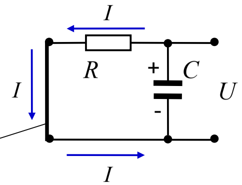
exponential charging

$U = U_T \left(1 - e^{-\frac{t}{\tau}}\right)$

At t=0 the capacitor is uncharged ($U = 0$). (In this moment the charging current is maximal: $I = U / R$.)

time constant

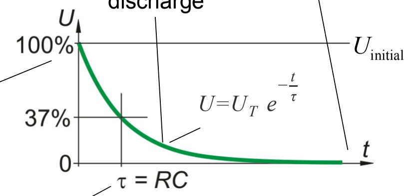
Discharging a capacitor through a resistor



short circuit ($R = 0$)

After a long time the capacitor loses the stored charge, the voltage on it becomes $U = 0$, and the discharge current stops.

At the first moment of the discharge the voltage on the capacitor is $U = U_{initial}$, the current is maximal $I = U_T / R$.



exponential discharge

$U = U_T\, e^{-\frac{t}{\tau}}$

time constant

Capacitive reactance

Reactance is the opposition of a circuit element to a change of current, caused by the build-up of electric or magnetic fields in the element.

symbol: X_C unit: ohm, | $X_C = U / I_C$

Although capacitive reactance is analogous to the resistance, no heat is produced on an ideal capacitor because $R = \infty$, and $P = U^2/R = 0$.

$X_C \sim 1/f$ $X_C \sim 1/C$ $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f \cdot C}$

If $f = 0$ (DC case), $X_C = \infty$ ⟶ The capacitor can be substituted with a breakage.

If $f = \infty$, $X_C = 0$ ⟶ The capacitor is equivalent with a short circuit.

Inductive reactance

symbol: X_C unit: ohm, | $X_L = U / I_L$

Although inductive reactance is analogous to the resistance, no heat is produced on an ideal coil because $R = 0$, and $P = I^2 R = 0$.

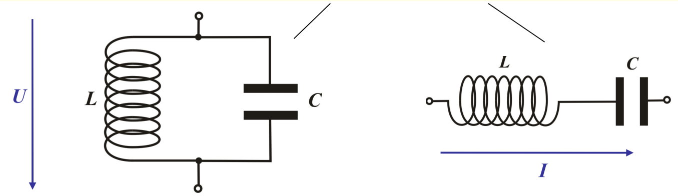
$X_L \sim f$ $X_L \sim L$ $X_L = \omega \cdot L = 2\neq f \cdot L$

If $f = 0$ (DC case), $X_L = 0$ ⟶ The coil can be substituted with a short circuit.

If $f = \infty$, $X_L = \infty$ ⟶ The capacitor is equivalent with a breakage.

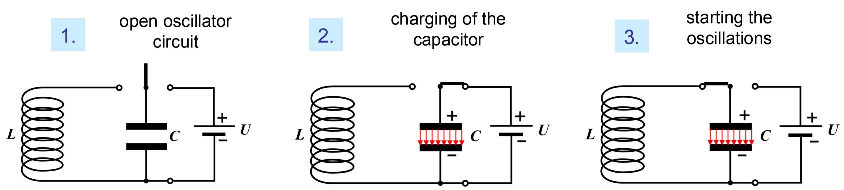
Oscillator circuit

The oscillator circuit consists of a coil and capacitor connected parallel or in series.



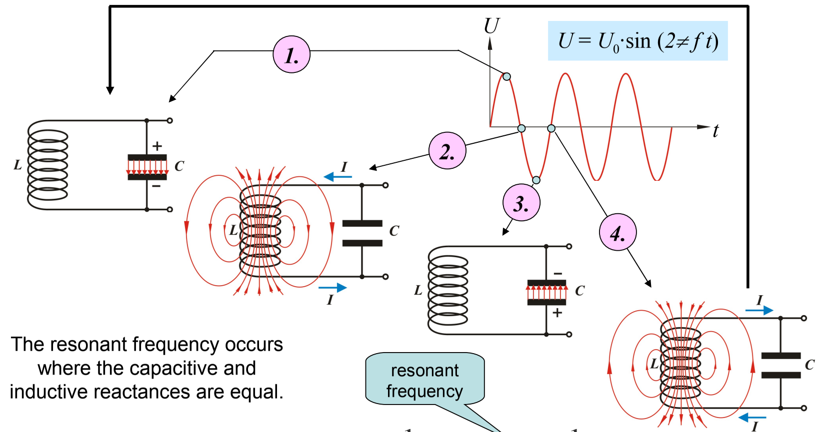
To start the oscillation we provide energy in the capacitor or in the coil.

The example below shows the start of a parallel oscillator circuit.



Oscillator circuit

The current and the voltage of a released ideal oscillator circuit will follow sinusoidal oscillations.



The resonant frequency occurs where the capacitive and inductive reactances are equal.

$X_L = X_C$ ⟶ $2\pi f L = \frac{1}{2\pi f C}$ ⟶ $f = \frac{1}{2\pi\sqrt{LC}}$