Geometric optics

Fermat principle. Light reflection and refraction on planar and curved surfaces, medical optical devices, geometrical optics of the human eye.

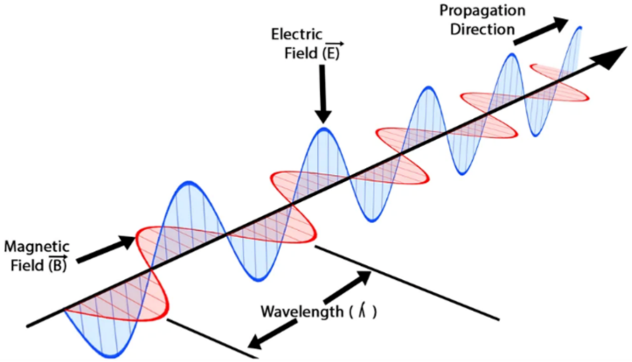
Erika Balog

Why?

- endoscopy, refractometry
- light microscope, stereo microscope
- optics of the eye

How?

Light – electromagnetic wave



Propagation of light

Geometric optics:

if: size of the object >> λ

light ray (light beam)

Wave optics:

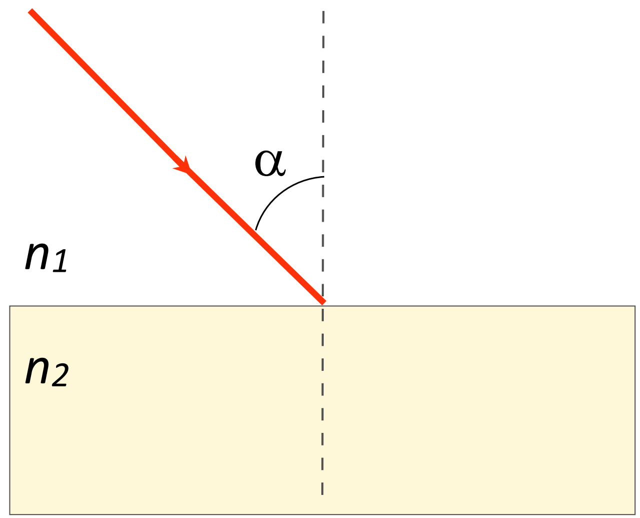
if: size of the object ~ λ

wave

Geometric optics

light ray (light beam): 

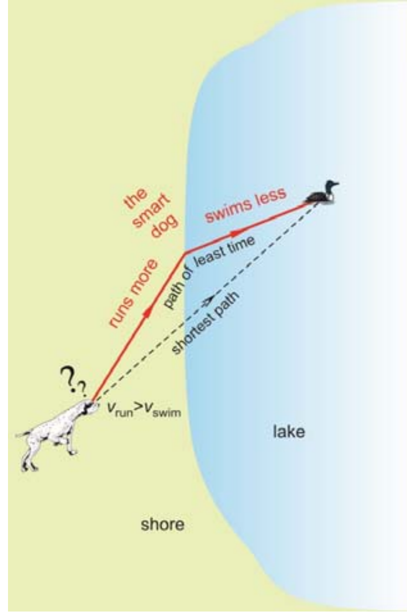
c_{vacuum}= 2,9979 x 10^8 m/s



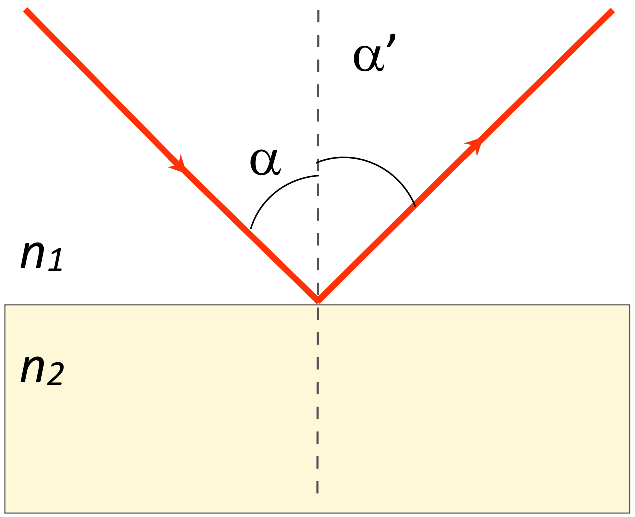
$$n_1 = \frac{c_{vacuum}}{c_1}$$

Fermat's principle – principle of the least time:

light follows the path that can be covered in the least time



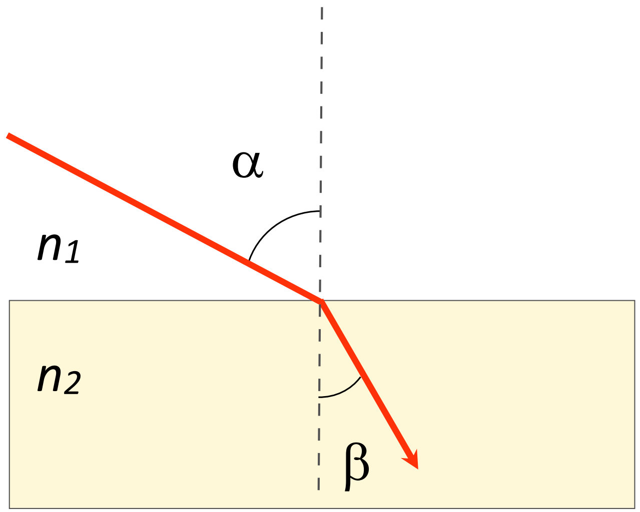
Reflection



- the incident and reflected beams and the axes of incidence are in the same plane.

α=α'

Refraction



$n_2 > n_1$

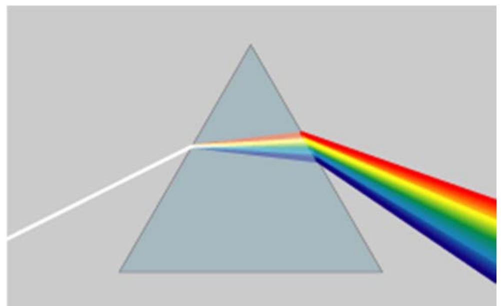
- the incident and refracted beams and the axes of incidence are in the same plane.

Snell's law:

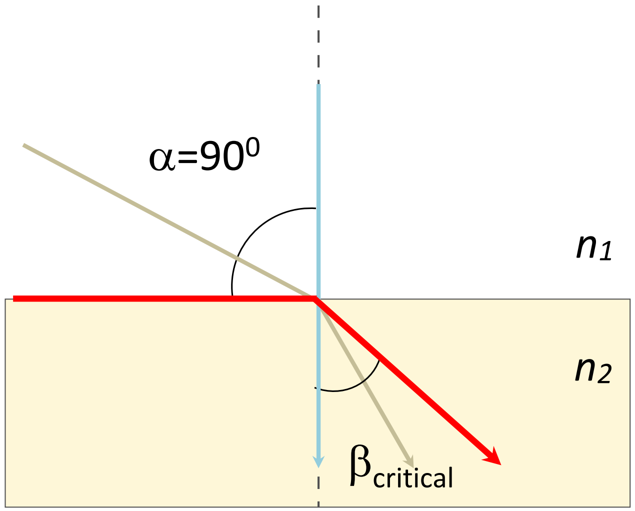
$$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2} = \frac{n_2}{n_1} = n_{21}$$

$$n_1 \sin\alpha = n_2 \sin\beta$$

Dispersion: the index of refraction depends on the wavelength



Critical angle (I)



$n_2 > n_1$

Snell's law:

$$n_1 sin\alpha = n_2 sin\beta$$

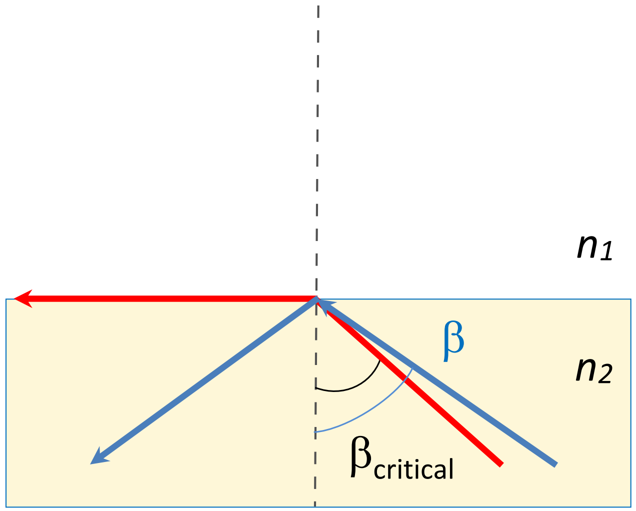
$$n_1 sin(90^0) = n_2 sin\beta_{critical}$$

$$sin(90^0) = 1$$

$$n_1 = n_2 sin\beta_{critical}$$

n_2 and $\beta_{critical}$ known ⟶ n_1

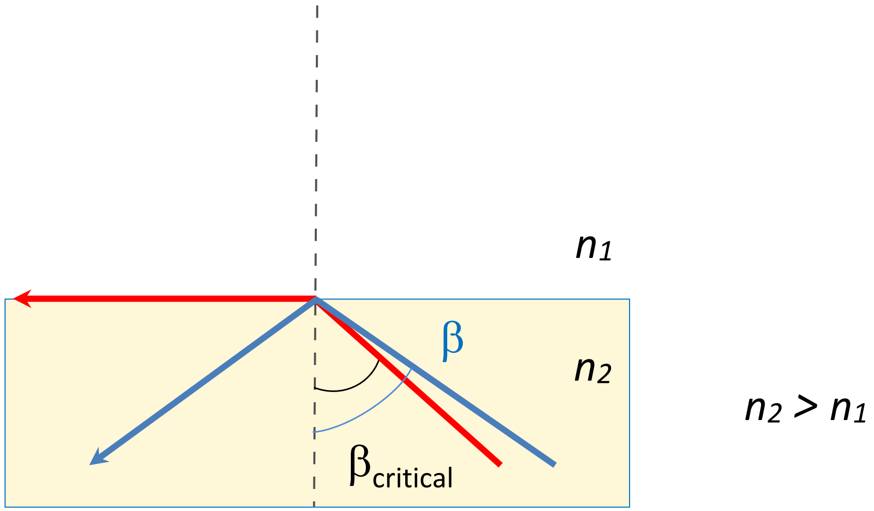
Critical angle – total reflection (II)



Principle of reversibility: the direction of propagation (arrows) may be reversed.

if: $\beta > \beta_{critical}$ ⟶ total reflection

Critical angle – total reflection (II)

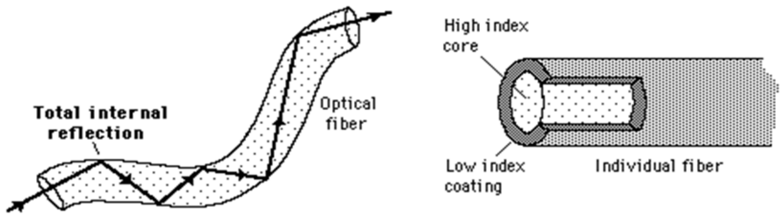


Principle of reversibility: the direction of propagation (arrows) may be reversed.

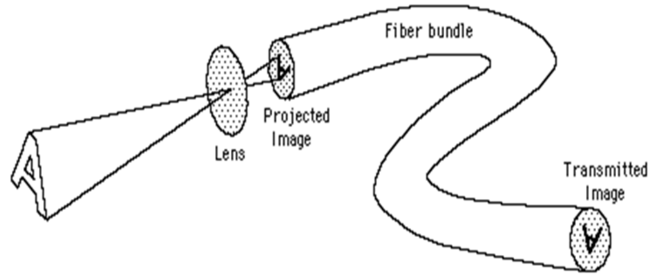
if: $\beta > \beta_{critical}$ $\longrightarrow$ total reflection

Application: optical fibers

Single fiber



Fiber bundle



- if the arrangement of the fibers is maintained within the bundle, then the image is faithfully transmitted.

Critical angle – total reflection (III)

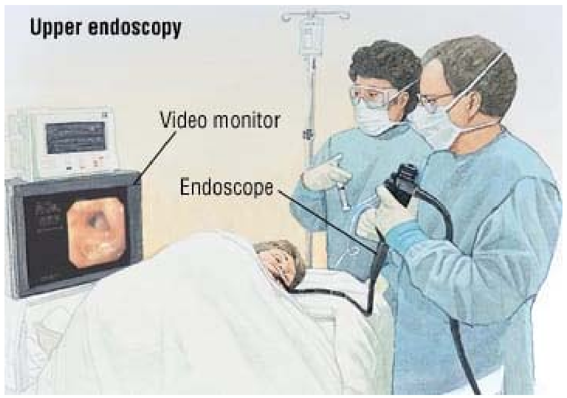
Medical application: *endoscopy*

OBJECTIVES

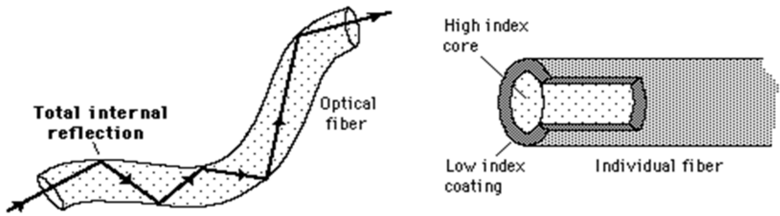
1. Diagnostics: visual inspection, biopsy, contrast agent delivery
2. Therapy: surgery, cauterization, removal of foreign objects

TYPES

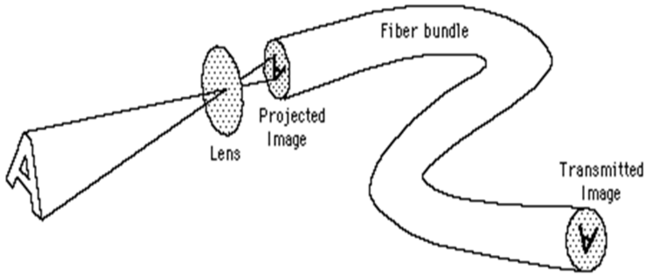
Arthroscopy (joints); *Bronchoscopy* (trachea and bronchi); *Colonoscopy* (colon); *Colposcopy* (agina and cervix); *Cystoscopy* (urinary bladder, urethra uterus, prostate via urethra); *ERCP* (endoscopic retrograde cholangio-pancreatography, delivery of X-ray contrast agent into biliary tract and pancreatic duct) ; *EGD* (Esophago-gastroduodenoscopy, upper GI tract); *Laparoscopy* (stomach, liver, female gonads via abdominal wall); *Laryngoscopy* (larynx); *Proctoscopy* (rectum, sigmoidal colon); *Thoracoscopy* (pleura, mediastinum and pericardium via chest wall).



Single fiber

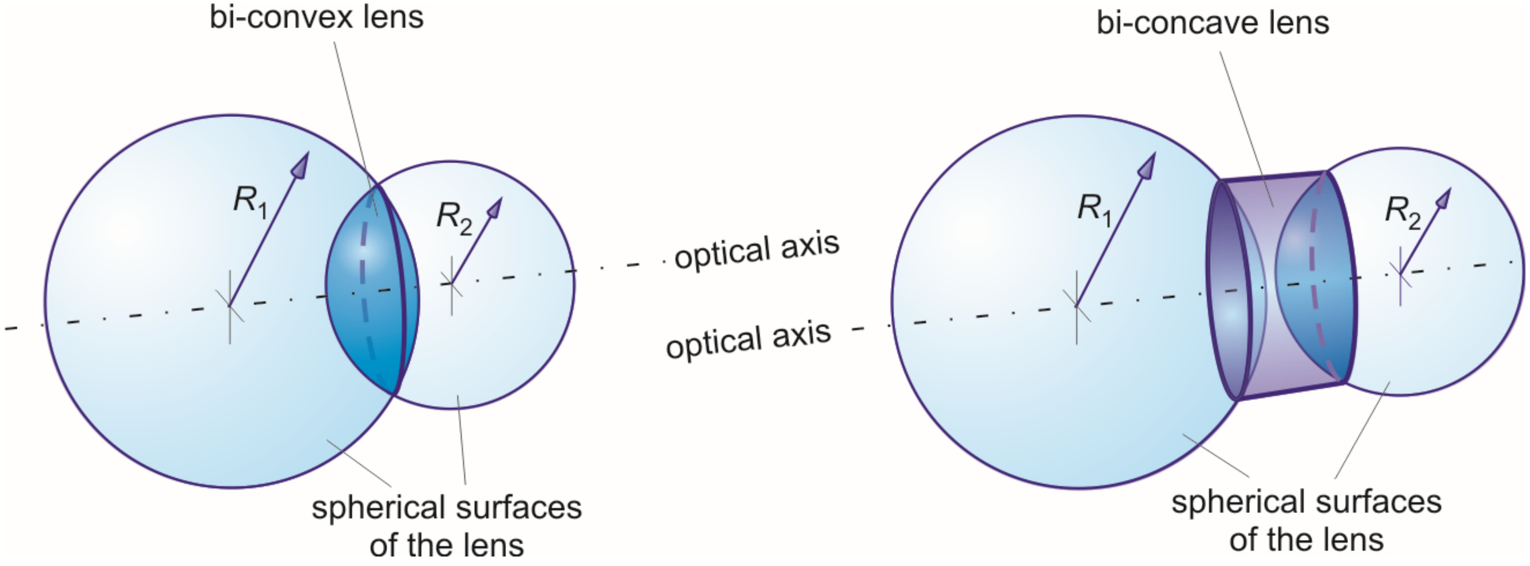


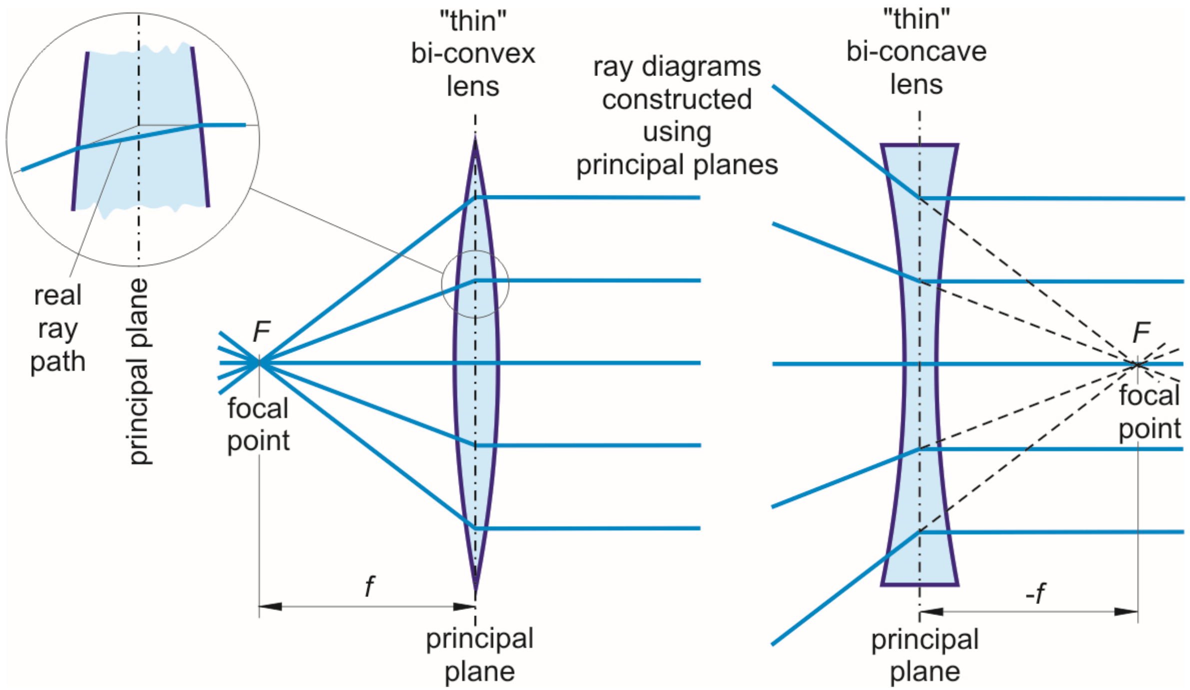
Fiber bundle



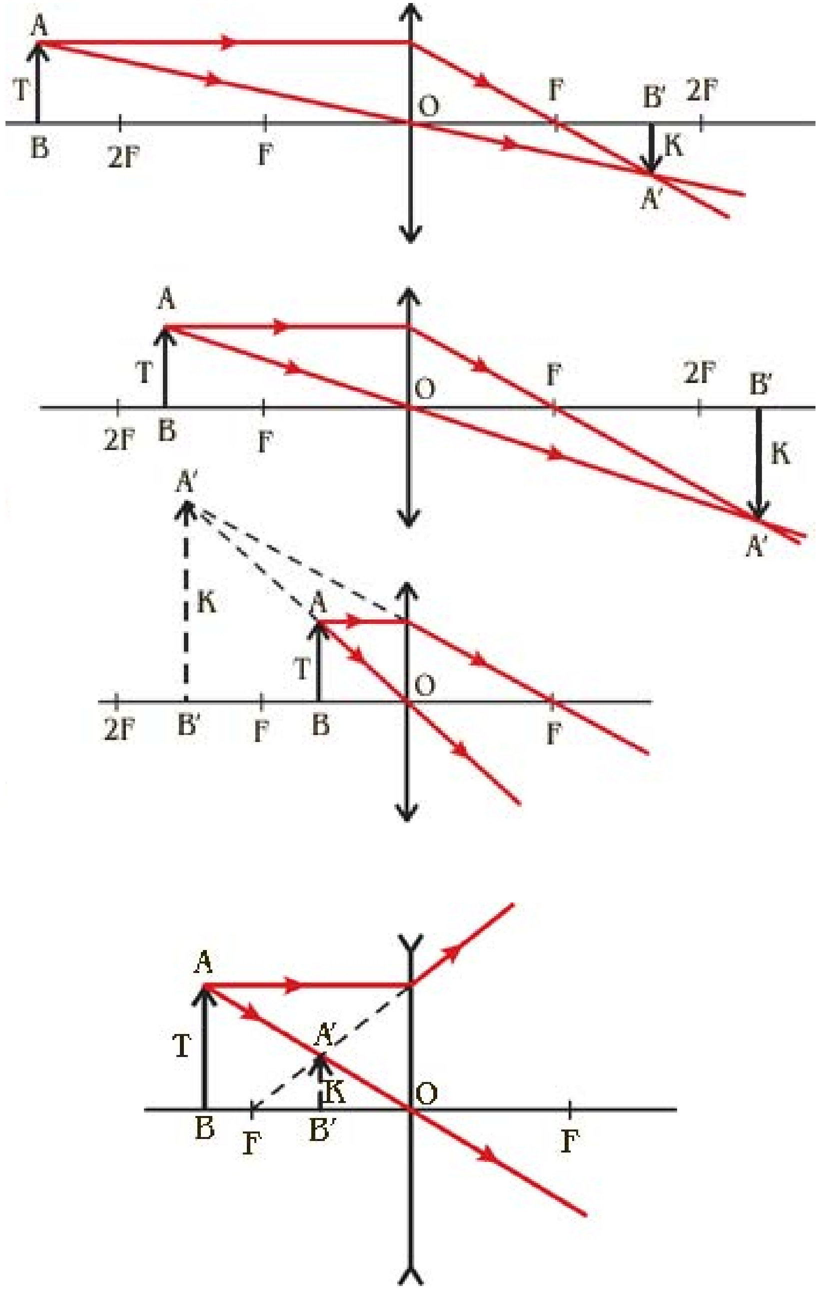
- if the arrangement of the fibers is mantained within the bundle, then the image is faithfully transmitted.

Optical lenses: refracting materials delimited by spherical surfaces

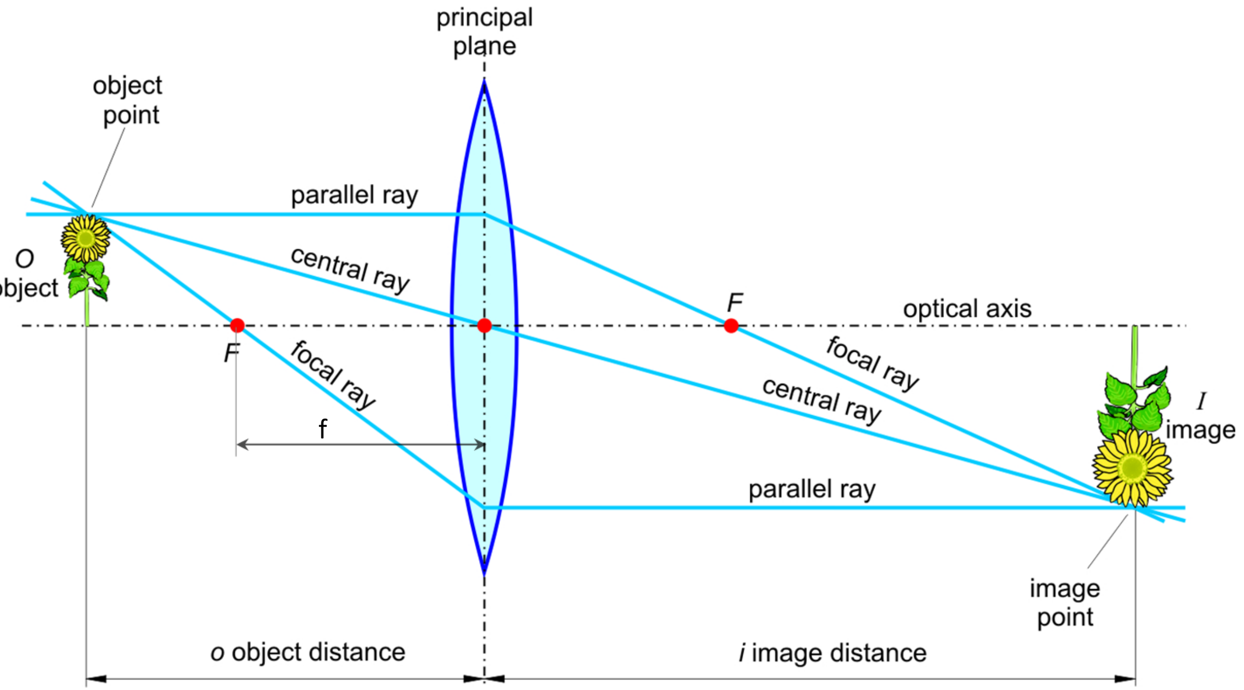




Optical lenses: imaging – principal rays



Optical lenses: imaging – lens equation



The lens equation $$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

Magnification $$M = \frac{I}{O} = \frac{i}{o}$$

D – optical power (diopter, m^{-1}) $$D = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

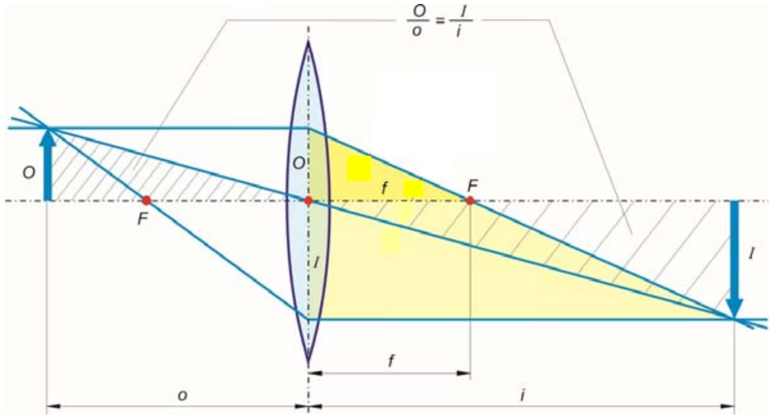
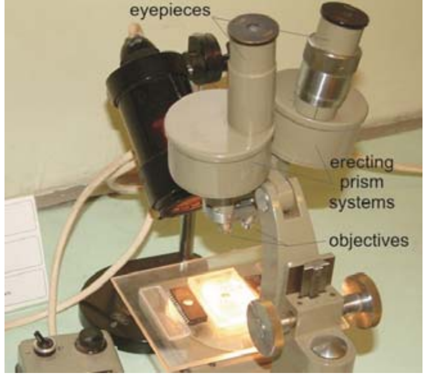
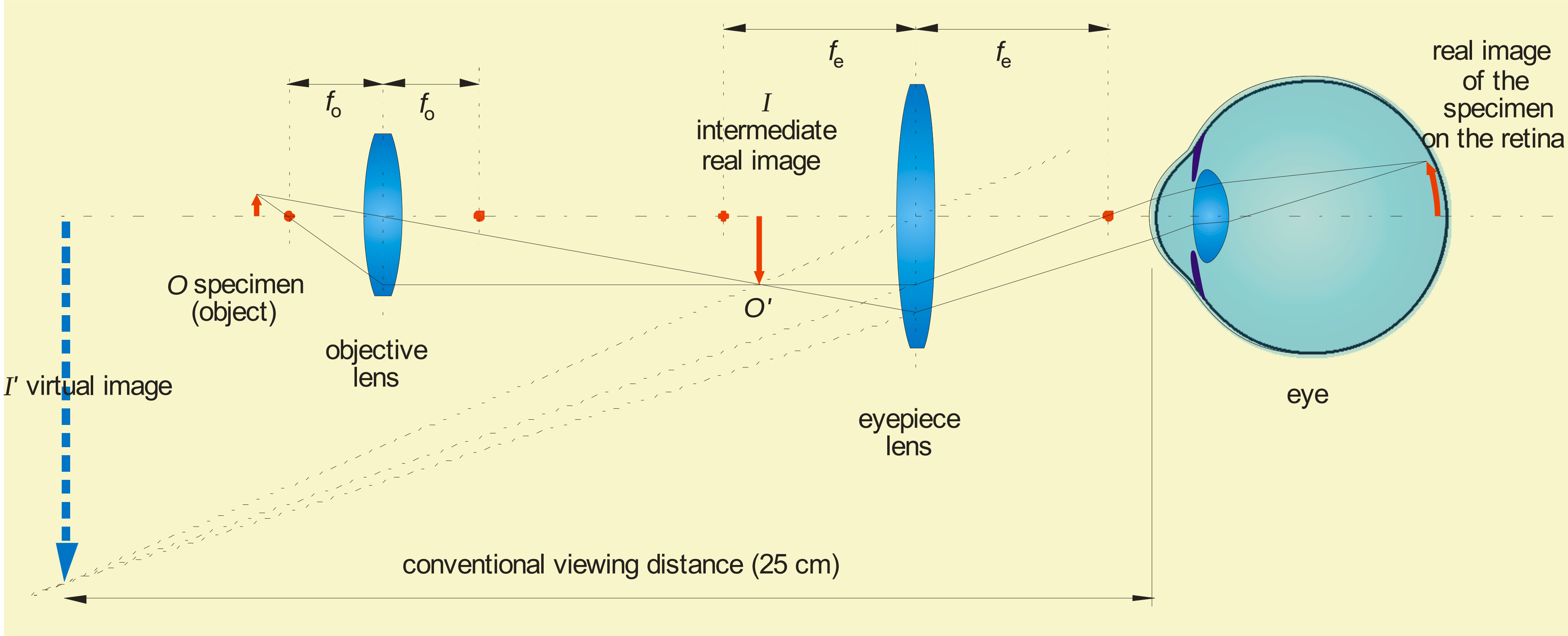


Image formation of the microscope

Stereomicroscope:

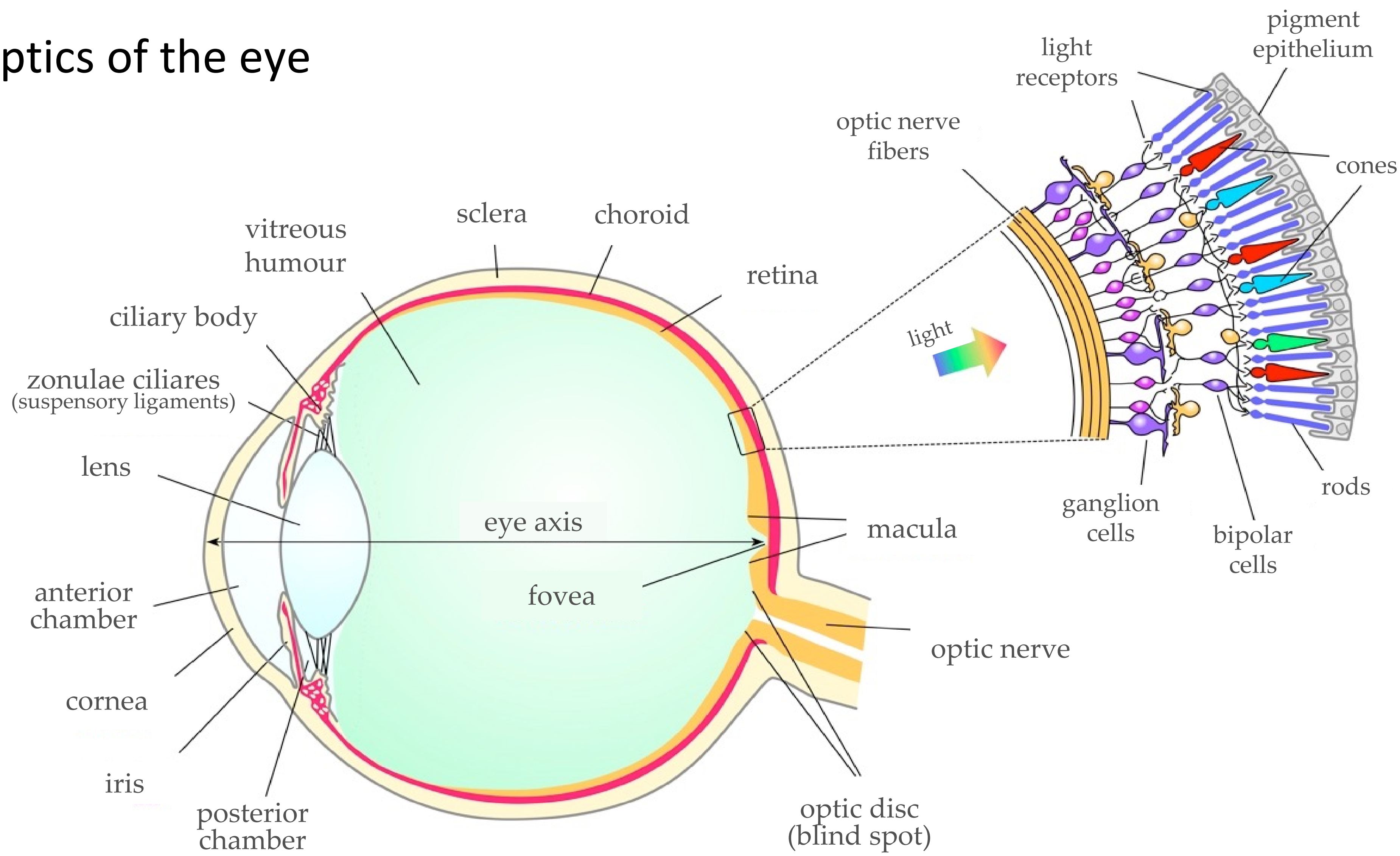




Magnification of the microscope:

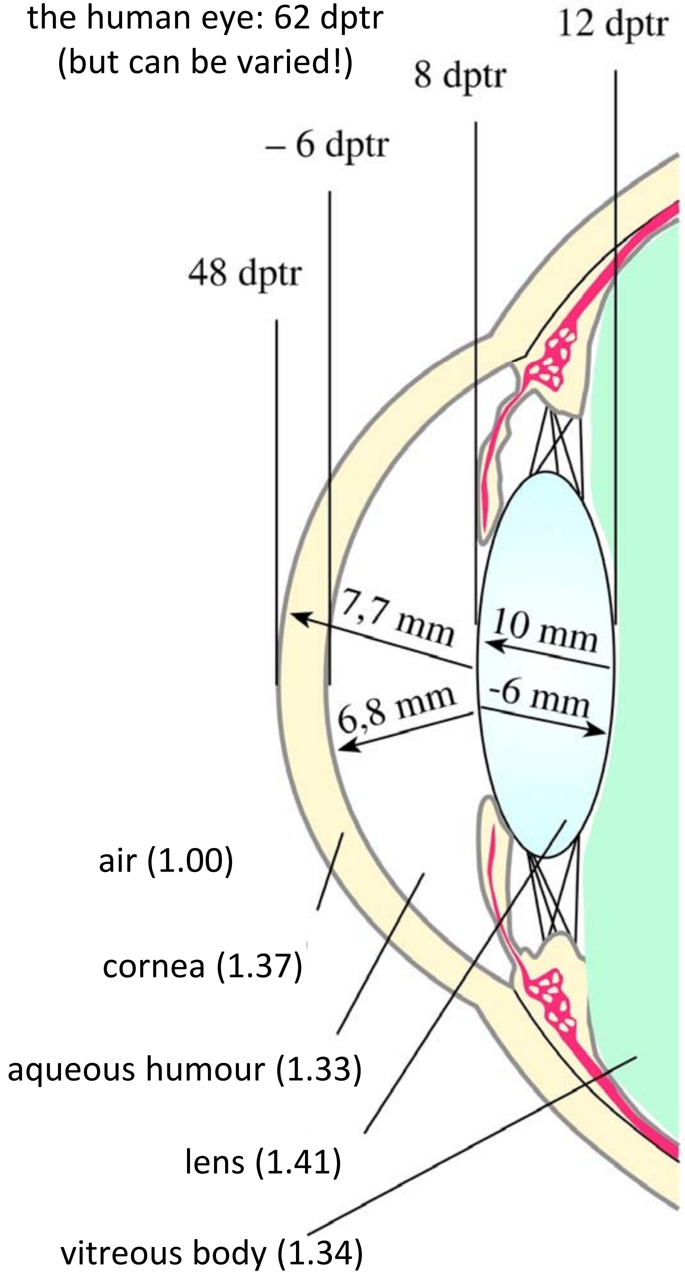
$$M_{micr} = \frac{I'}{O} = \underbrace{\frac{I'}{O'}}_{M_{eyep}} \cdot \underbrace{\frac{O'}{O}}_{M_{obj}} = M_{obj} \cdot M_{eyep}$$

Optics of the eye



Horizontal section of the human eye

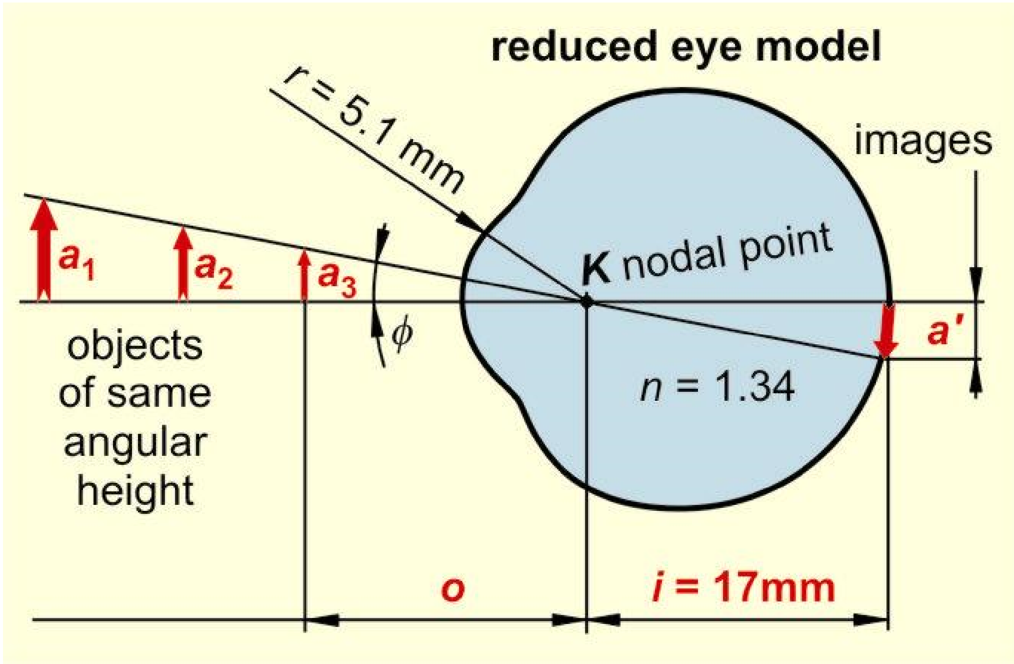
Total refractive power of
the human eye: 62 dptr
(but can be varied!)



Refractive power of surfaces (*D*, dptr):

$$D = \frac{n - n'}{R}$$

n-n' - refractive index difference of bounding media (air, cornea, etc.).
R - radius of curvature of refractive surface.



Inverted, diminished image is formed on the retina.

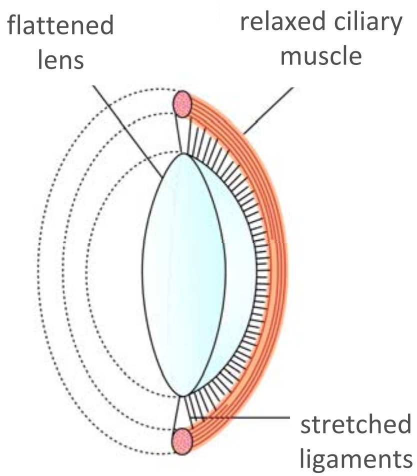
Accommodation: adaptation of the eye's refractive power to the object distance.

$$D = \frac{1}{o} + \frac{n'}{i} \qquad\qquad D = \frac{1 - n'}{R}$$

$$\frac{1 - n'}{R} = \frac{1}{o} + \frac{n'}{i}$$

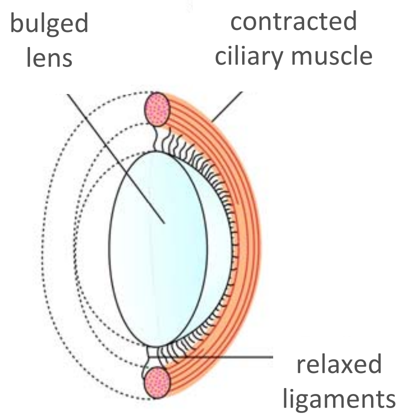
Farsight:

o increases $\longrightarrow$ *D decreases* $\longrightarrow$ R increases



Nearsight:

o decreases $\longrightarrow$ *D increases* $\longrightarrow$ R decreases



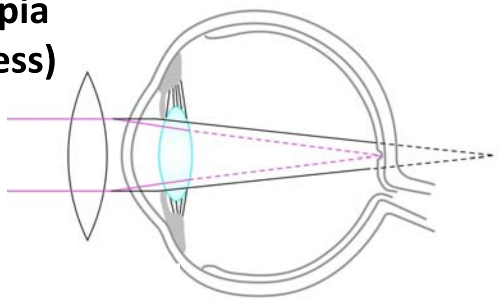
Accommodation power:

$$\Delta D = D_p - D_r = \frac{1}{o_p} + \frac{1}{o_r}$$

o_p – near point vision
o_r – far point vision

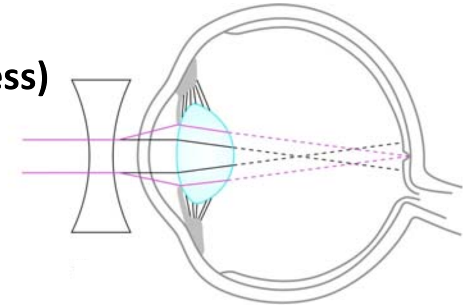
Refraction problems:

Hypermetropia (farsightedness)



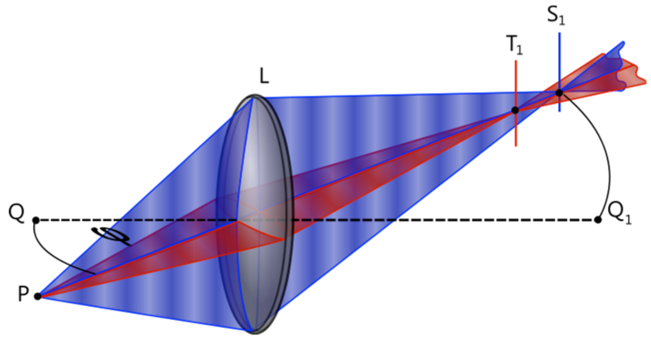
- shortened eyeball.
- correction with convergent lens.

Myopia (nearsightedness)



- elongated eyeball.
- correction with divergent lens.

Astigmatism: focal distance is different in the horizontal and vertical plane.

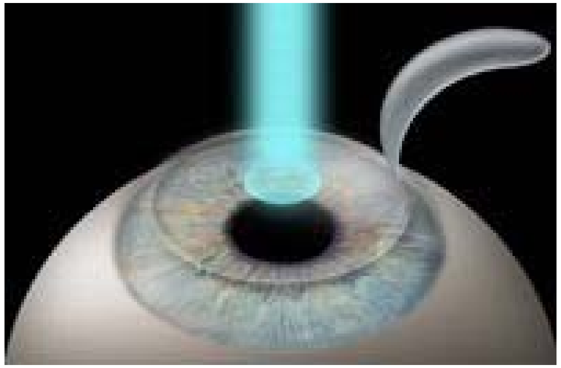


Correction with cylindrical lens.

Presbyopia:

- Accommodation power decreases (R!).
- Manifests with age (>45 years).
- Nearsight worsens.

Permanent correction of refractive problem: LASIK (Laser Assisted In Situ Keratomileusis)



The radius of curvature of the cornea is changed (with laser surgery).