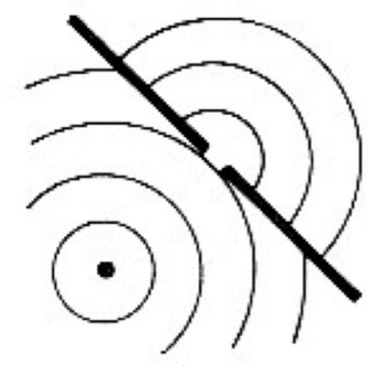
Wave optics

Miklós Kellermayer

Wave optics

If light propagates through a slit comparable or smaller than its wavelength, then its wave properties ust be taken into account.

Some phenomena cannot be explained with geometric optics



Important parameters of the propagating wave:

- **Period** (T)
- **Frequency** ($f=1/T$)
- **Velocity** (v, c)
- **Wavelength** (λ): distance covered in a period:

$$\lambda = cT = \frac{c}{f}$$

Speed of propagation of light in ***vacuum***: $c=2.99792458 \times 10^{8}\ ms^{-1}$

Wave: propagating oscillation

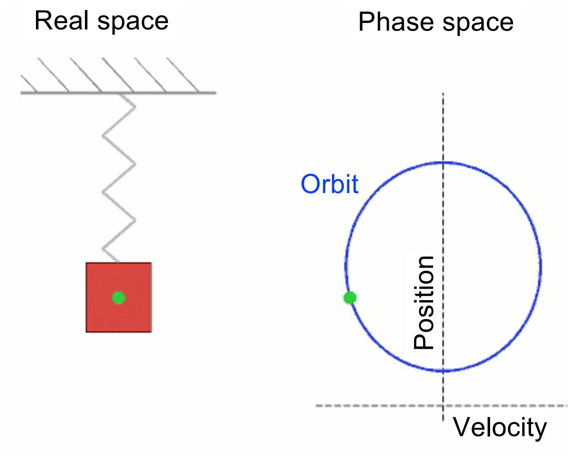
Example: Mechanica l oscillation

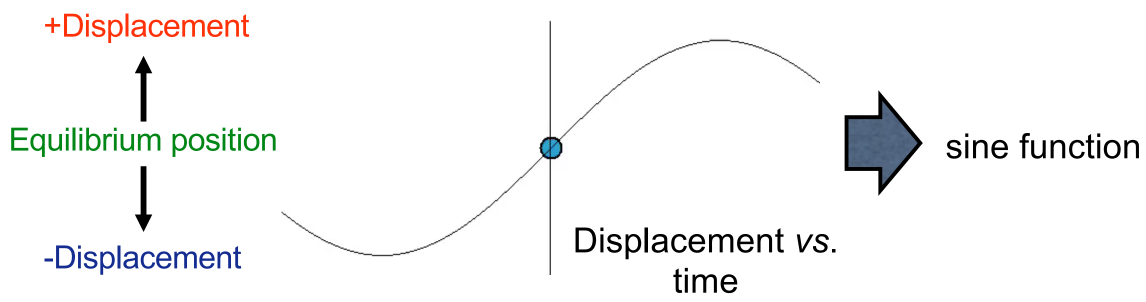
Tacoma Narrows Bridge (“Gallopin’ Gertie”, *“Gertie the Dinosaur”* (1914), cartoon)

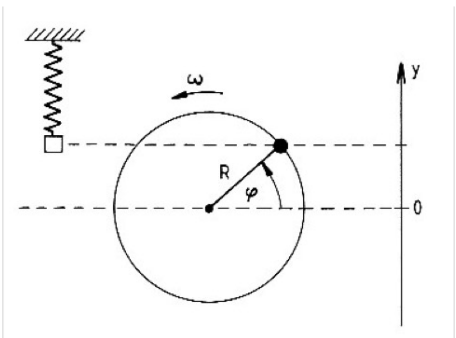
- Opening: July 1, 1940.
- During wind (50-70 km/h): oscillation for hours
- Oscillation amplitude initially 0.5 m, then, after snapping of a suspension cable, up to 9 m!
- Collapse: November 7, 1940.

Harmonic oscillation

Restoring force acts on a system displaced out of equilibrium (e.g., mass on a spring).







ϕ = phase angle at time t
y = displacement at time t
ω = angular velocity (ϕ/t)
R = length of rotating unit vector = maximal displacement (amplitude)

$$y = R\sin\varphi$$

Because $\phi = \omega t$:

$$y = R\sin(\omega t)$$

If the initial phase angle (ϕ_0) differs from 0:

$$y = R\sin(\omega t + \varphi_0)$$

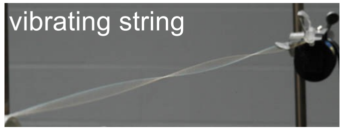
Because angular velocity (ω) is the full circular orbit (2π) per period (T):

$$y = R\sin(\frac{2\pi}{T}t + \varphi_0)$$

Types of waves

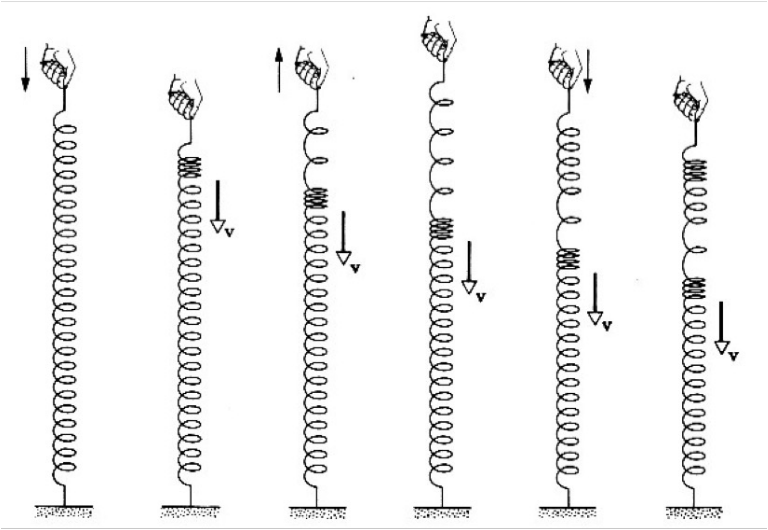
- According to source:
  1. Mechanical: elastic deformation propagating through elastic medium
  2. Electromagnetic: electric disturbance propagating through space (vacuum)

- According to propagation dimension:
  1. One-dimensional (rope)
  2. Surface waves (pond)
  3. Spatial waves (sound)

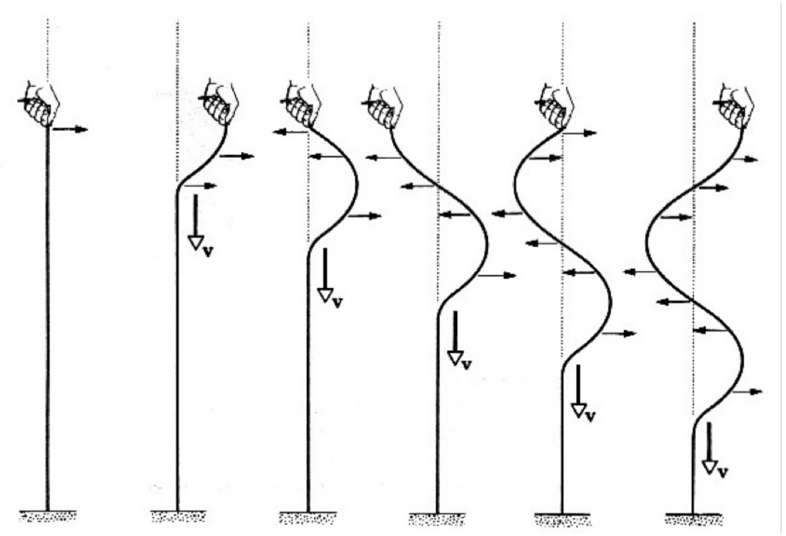


- According to relative direction of oscillation and propagation:

1. Longitudinal



2. Transverse



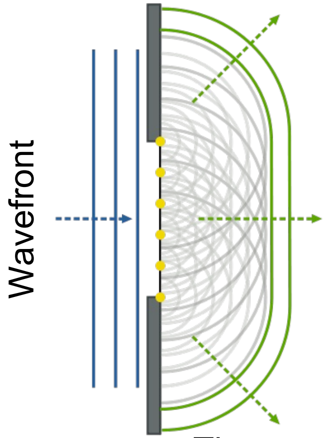
Wave phenomena I.

Diffraction

Huygens-Fresnel principle:
every point of a wavefront is the source of further waves

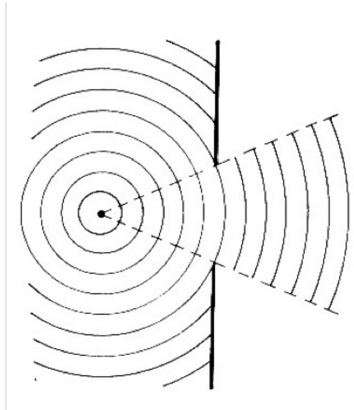
Christiaan Huygens (1629-1695)

Augustin-Jean Fresnel (1788–1827)

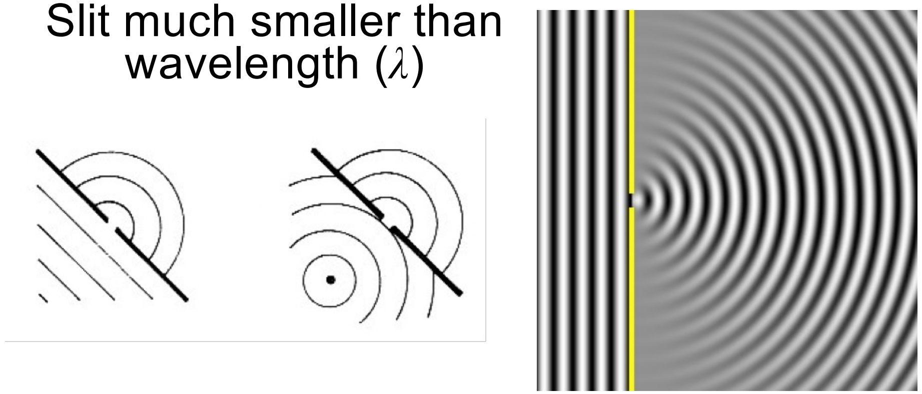


The wave appears in the "shaded" areas, too.

Slit much greater than the wavelength (λ)



Slit much smaller than wavelength (λ)



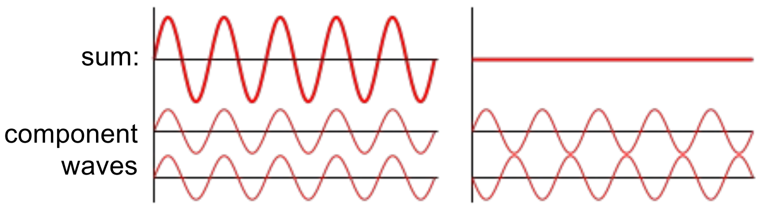
Wave phenomena II.

interference

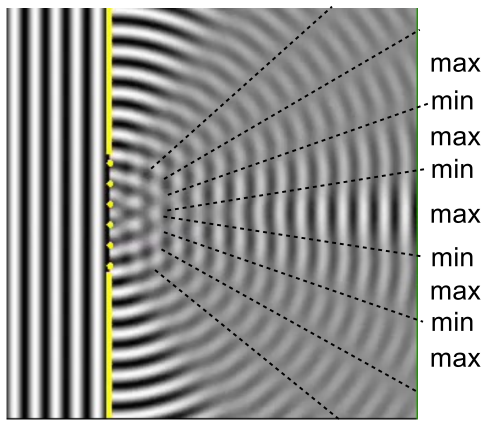
Principle of superposition

Waves in phase ($\Delta\varphi=0$): amplification

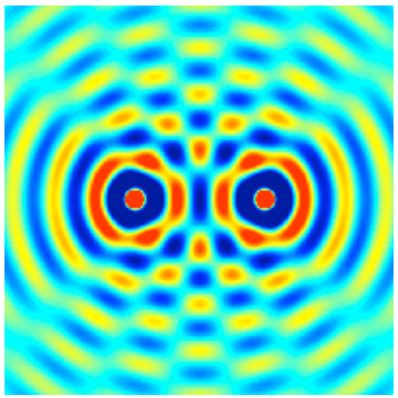
If $\Delta\varphi=\pi$: cancellation



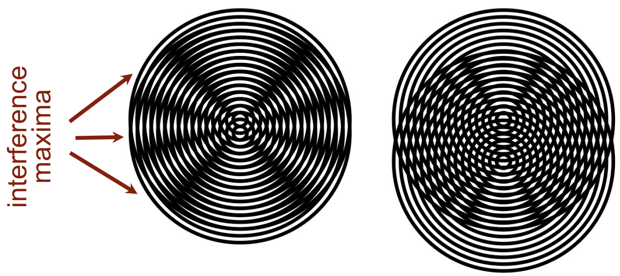
Slit comparable to wavelength (=pointlike slits separated by distance d, where $d \sim \lambda$)



Interference of waves emerging from two point sources.



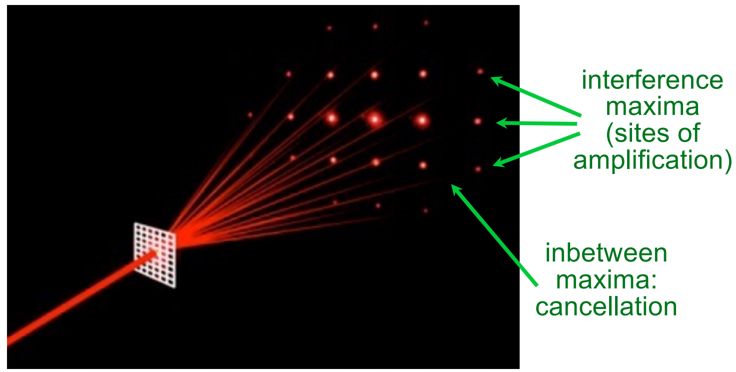
Interference pattern depends on distance (d) separating the pointlike slits



small d ~ large distance between maxima

large d ~ small distance between maxima

Diffraction pattern of a 2D optical grating

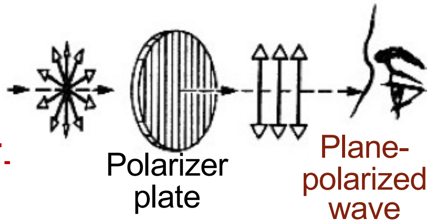


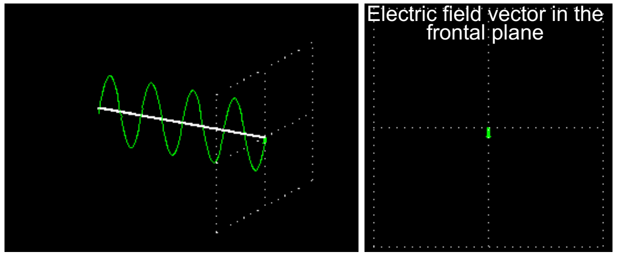
Wave phenomena III.

Polarization: oscillation in preferred direction

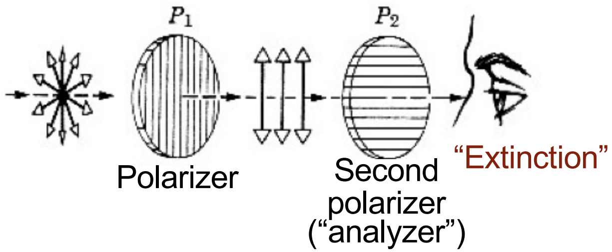
Only **transerse** waves may be polarized.

Plane or linear polarization.

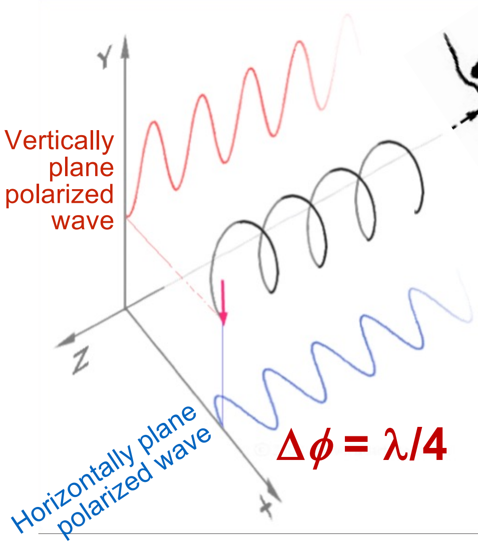


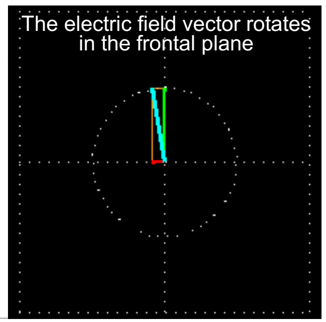


Plane-polarized light may be abolished by a second polarizer in crossed position

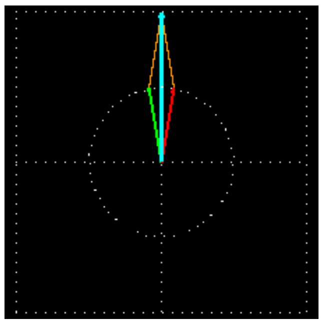


Circular polarization



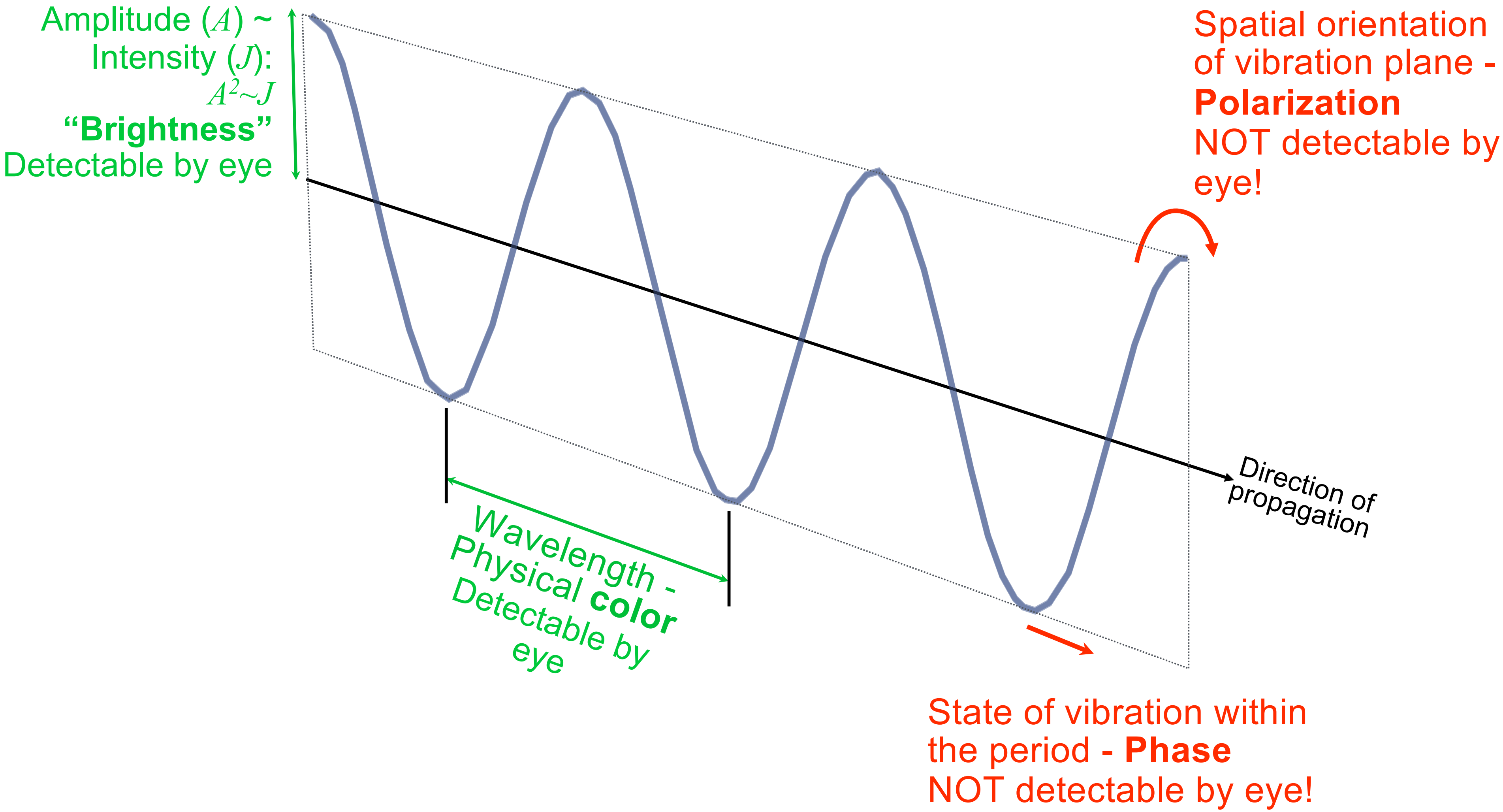


N.B.: Plane-polarized light — sum of right- and leftward rotating circularly polarized light



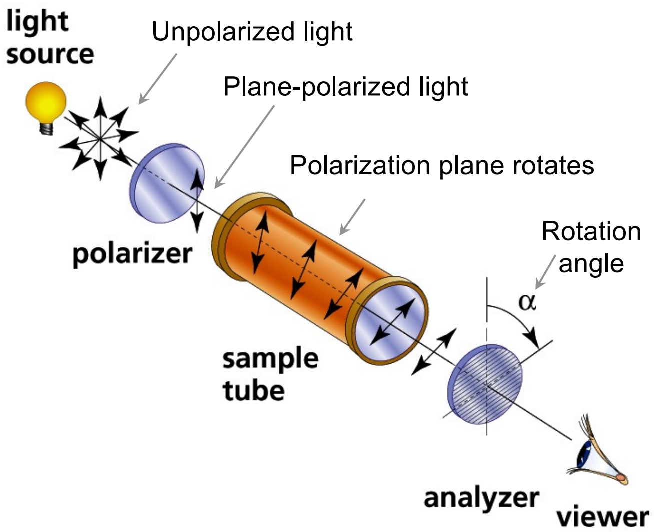
($\Delta\phi = 0$)

Detectable parameters of the light wave



Applications of polarization

Polarimetry



Rotation angle depends on the concentration (c) of the optically active* material:

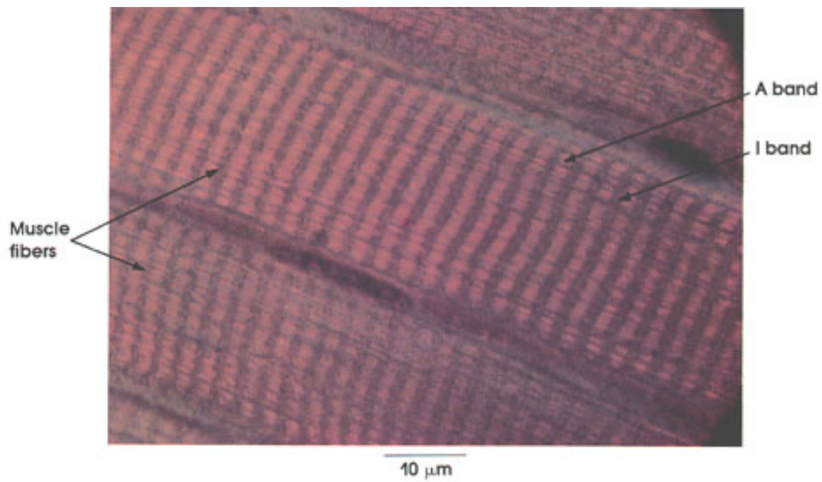
$$\alpha = [\alpha]_D^{20} \cdot c \cdot l$$

$[\alpha]$ = specific angle of rotation ("20": room temperature; "D": emission spectral line of Na λ=589 nm)

l = length of sample tube

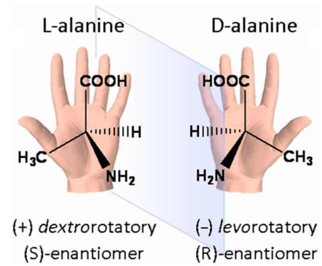
Polarization microscopy

Cross-striated skeletal muscle



- A-band: anisotropic (birefringent) region (contains ordered myosin molecules)
- I-band: isotropic region

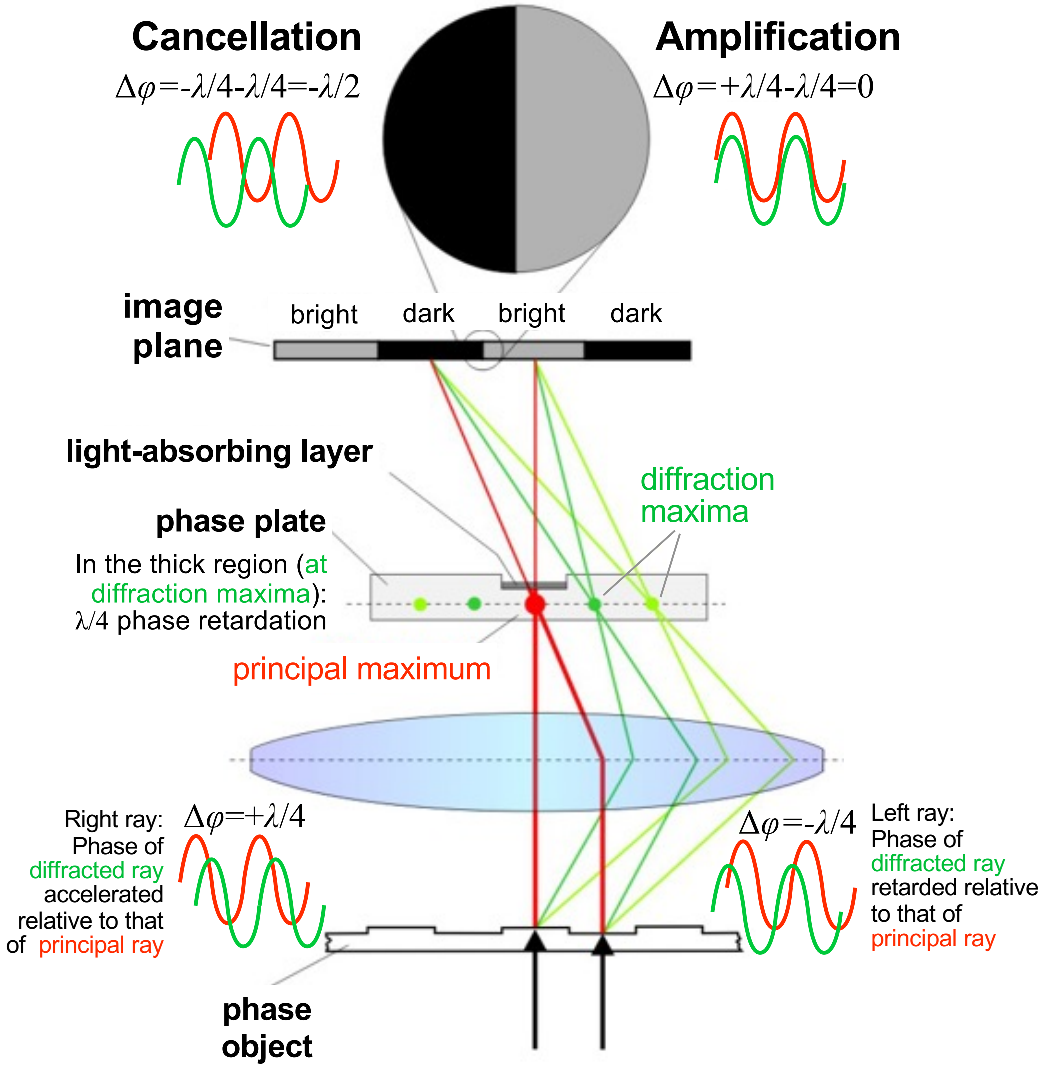
*Optically active material: contains ***chiral*** (mirror-symmetric) molecules that rotate the plane of polarization.



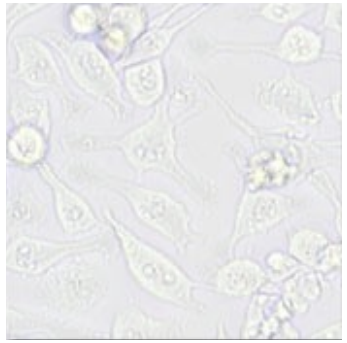
Phase contrast microscopy

Frits Zernike (1888-1966)
Nobel-prize

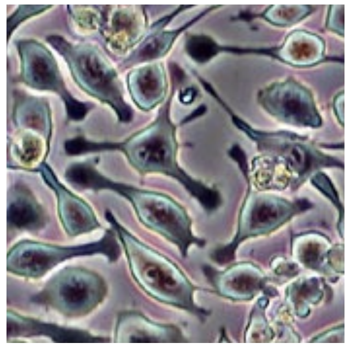
- Phase: shows the state of vibration within the entire period (2π).
- Expressed with the phase angle (φ).
- Phase difference between waves ($\Delta\varphi$): phase shift (retardation or acceleration)



Live (unstained) cells



Bright-field microscopic image

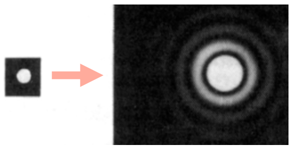


Phase-contast microscopic image

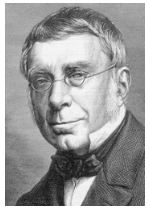
Resolution of the human eye I.

Diffraction limit

Because of diffraction: image of a point object is an Airy disk

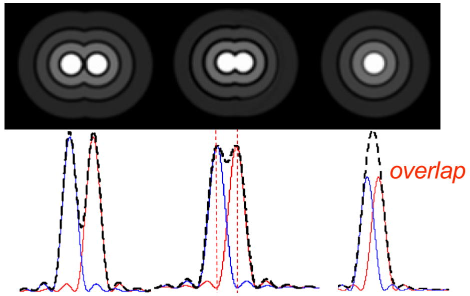


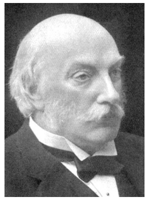
Object   Image



Sir George Biddel Airy (1801-1892)

Rayleigh criterion: objects may be resolved if their corresponding Airy disks do not overlap





Lord Rayleigh (1842-1919)

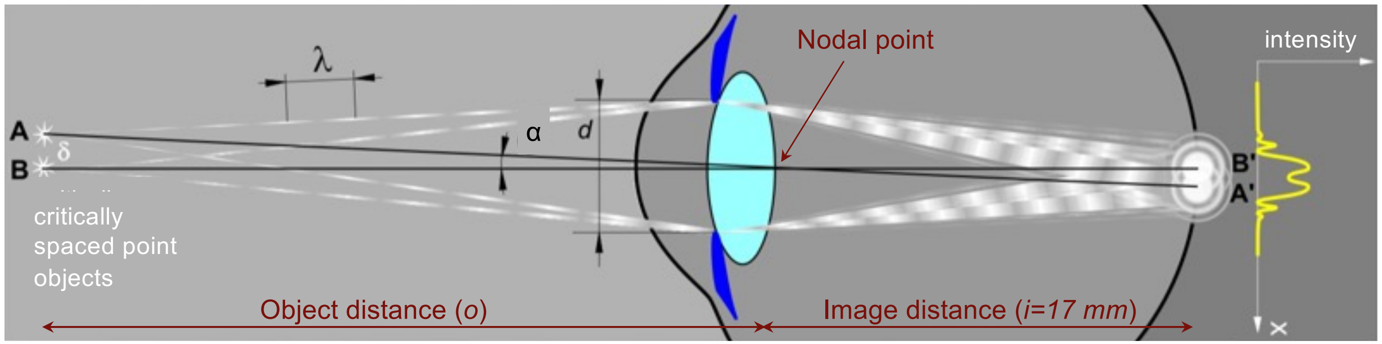
Smallest resolved distance has a limit (Abbe equation):

$$d = \frac{0.61\lambda}{n \sin\alpha}$$

λ = wavelength
n = refractive index of medium
α = angle between axis and outermost ray

Ernst Abbe (1840-1905)

Diffraction limit of the human eye

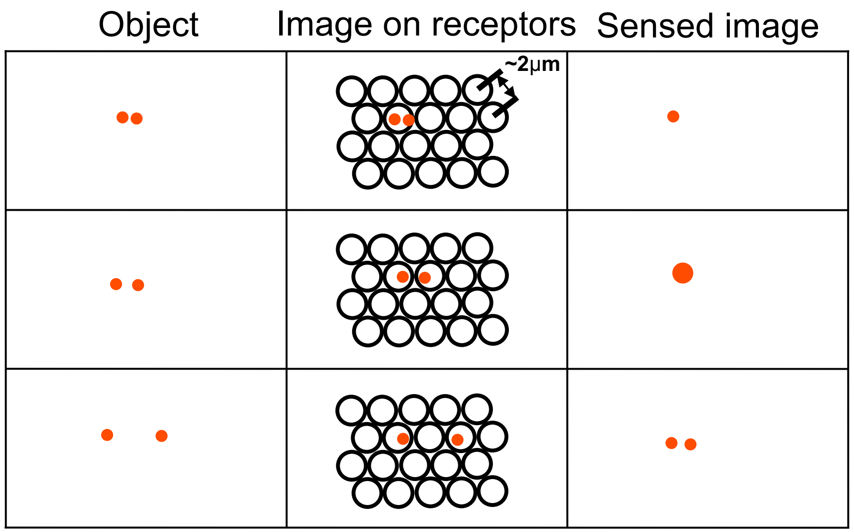


Reduced eye model

Limiting visual angle: $\alpha_H = 1.22\frac{\lambda}{d}$ Smallest angle of view at which two closely spaced objects may be resolved. At average wavelength (550 nm) and pupil diameter (4 mm): **0.6’** (angular minutes)

Resolution of the human eye II.

Biological limit: receptor cell density



- Condition of resolution: at least one inactivated receptor cell falls in between two activated ones. The limiting angle of view under this condition is (α_B)≈ 0.8'
- The diffraction and biological limits of the human eye are **comparable**!

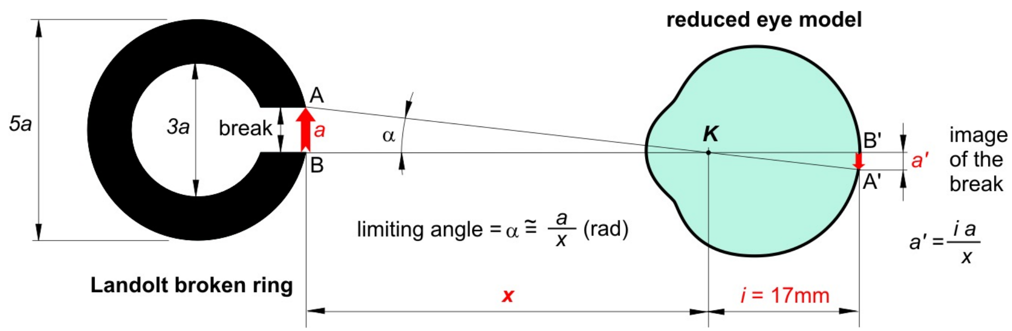
Visual Acuity ("visus", vision):

$$visual\_acuity = \frac{1'}{\alpha} 100\%$$

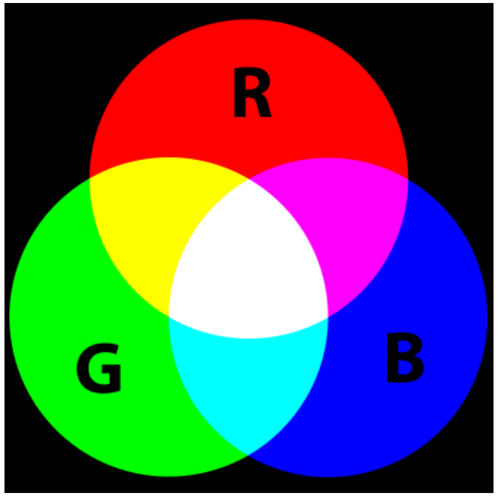
α = experimental (measured) visual angle

Average visual angle in healthy humans: 1' (= 100% vision)

Measurement of visual acuity



Color coding, color vision



Additive color coding

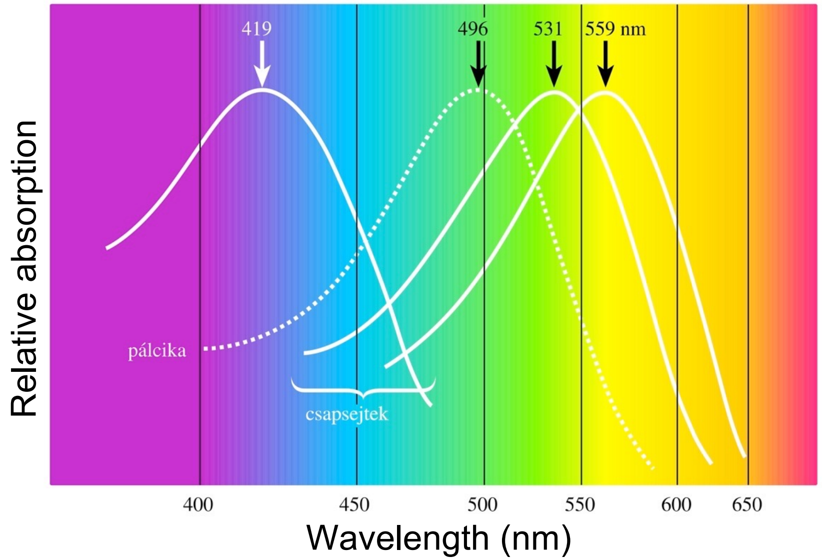
Any color may be generated by mixing three basic colors (R=red, G=green, B=blue) with varying weighing factors.

$$X = rR + gG + bB$$

In the human eye:

- 3 different color-sensitive receptors.
- Each receptor absorbs in different regions of the visible spectrum (R=64%, G=32%, B=2%).

Absorption spectra of the human color-sensitive receptors (cones)



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