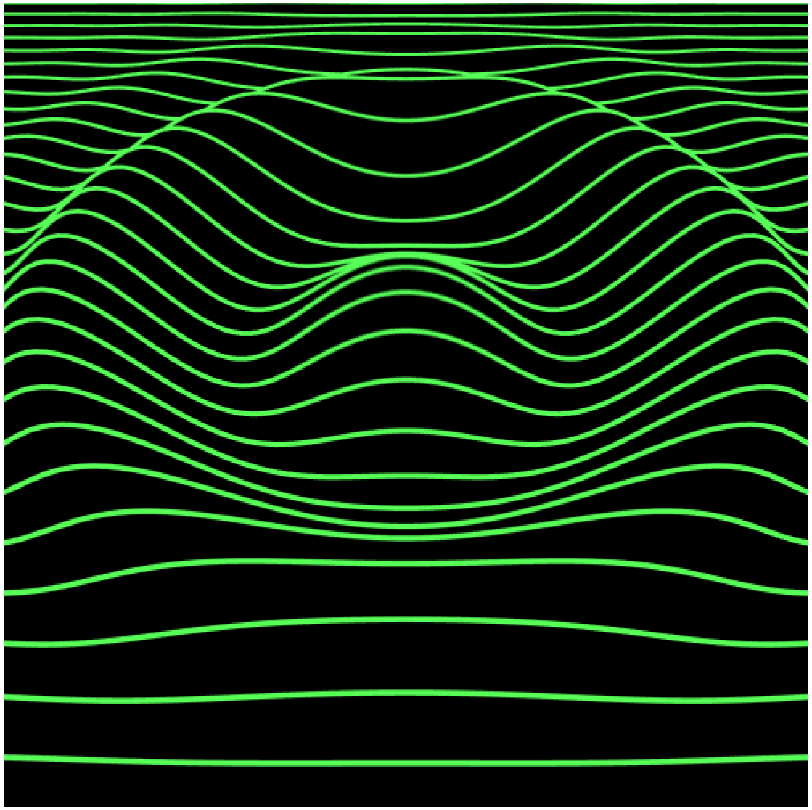
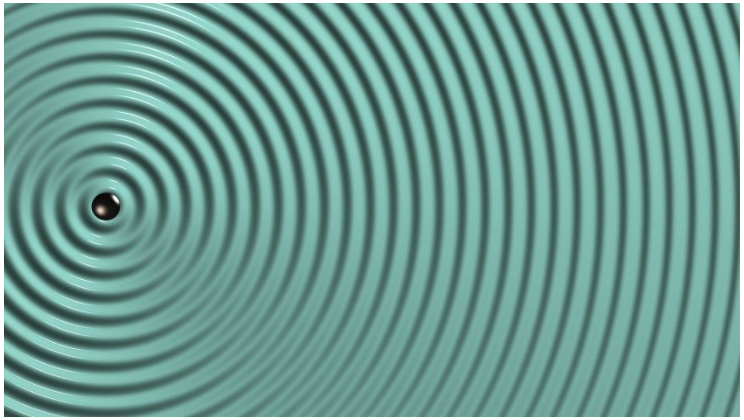
Wave optics

Waves are everywhere, not just in optics!

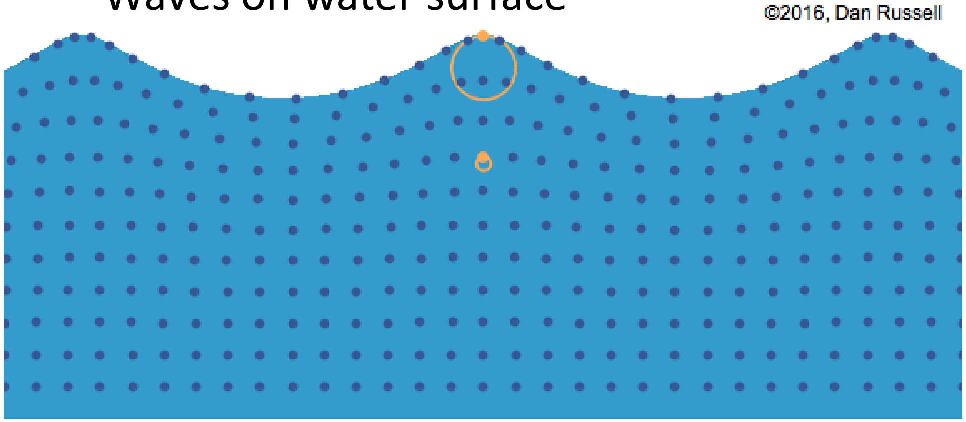




G.Schay

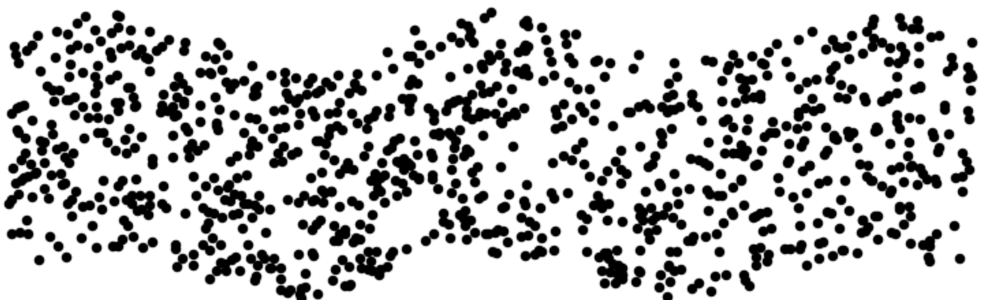
The individual particles oscillate locally, and only the "wavefront" is moving forward.

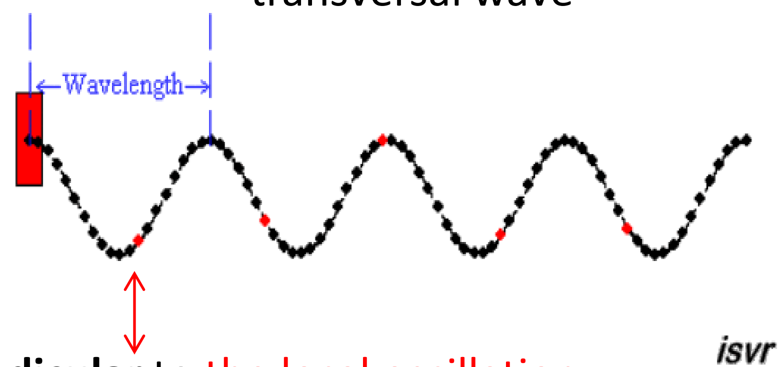
Waves on water surface



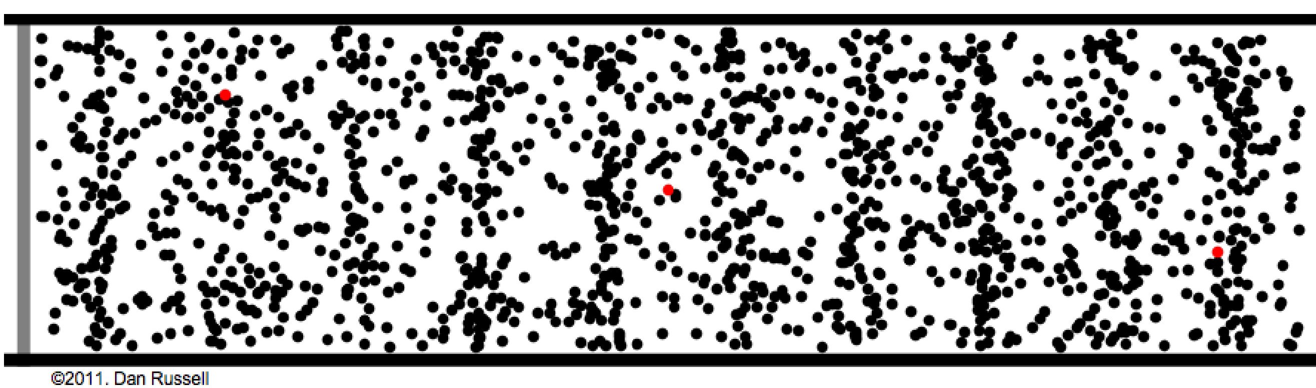
the "motion" of the wave is the propagation of the displacement of the particles

transversal wave





The propagation direction of the wave **is perpendicular** to the local oscillation



Longitudinal waves:
The propagation direction of the wave **is parallel** to the local oscillation

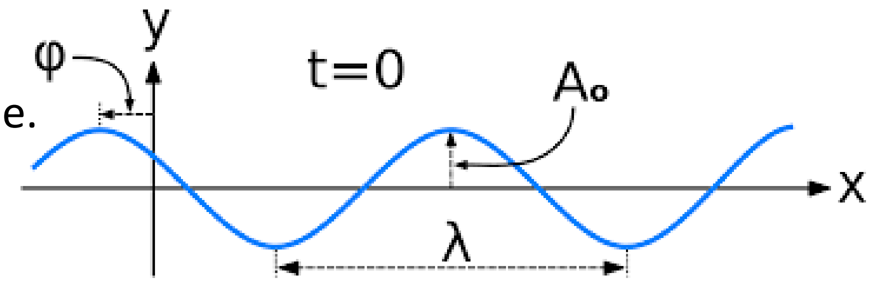
$y(x,t) = A * \sin(k*x + \omega*t + \phi)$

where A is the amplitude of the wave, k is the wavenumber, and ω is the angular frequency

$\omega = 2\pi f$, where $f = 1/T$ [Hz], while T is the period time.

$\omega = c*k$ defines the wavenumber, which can be written as $k = 2\pi/\lambda$, where λ is the wavelength.

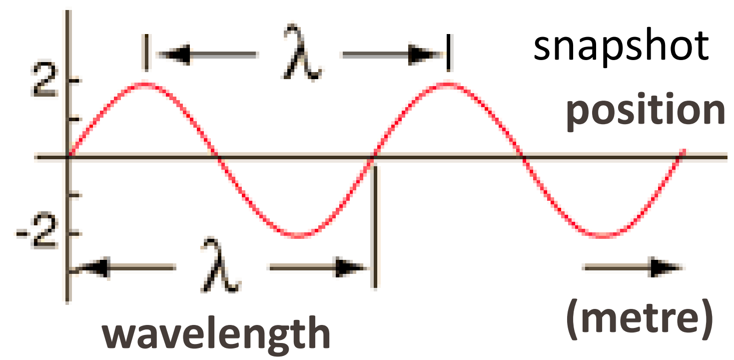
graph of a wave

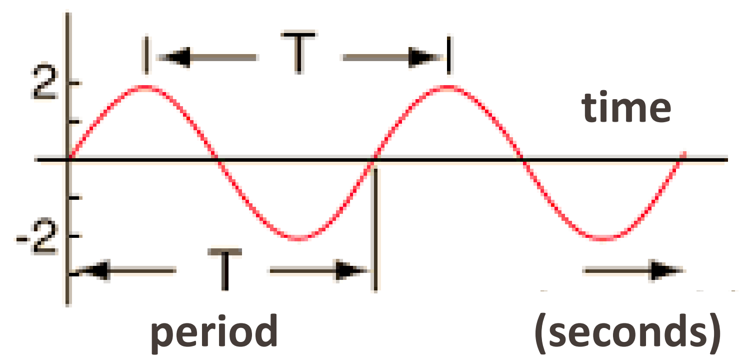


$$c = \frac{\lambda}{T} = \lambda \cdot f$$

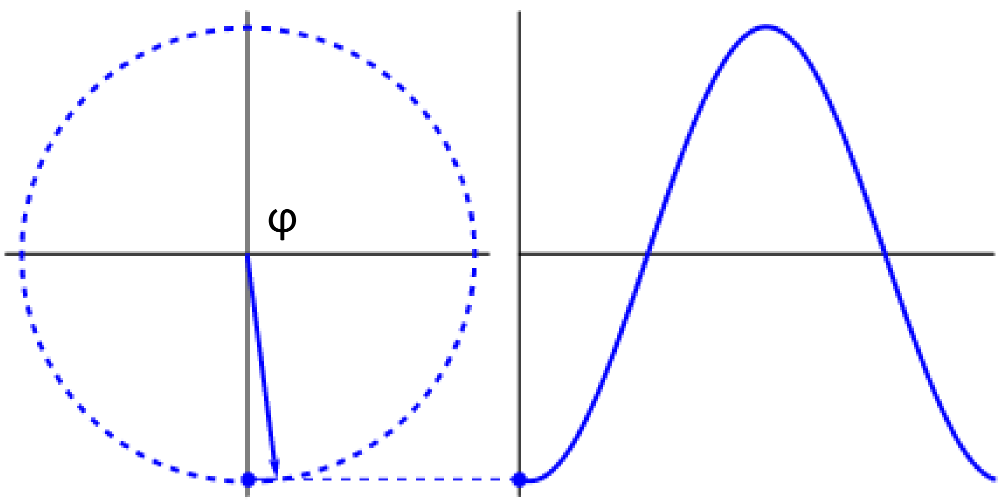
a harmonic wave is a sinusoidal function in space and time, and can be modelled with a circular motion. (a full circle is 2π in angle).

the **phase** is the position along the circle: φ

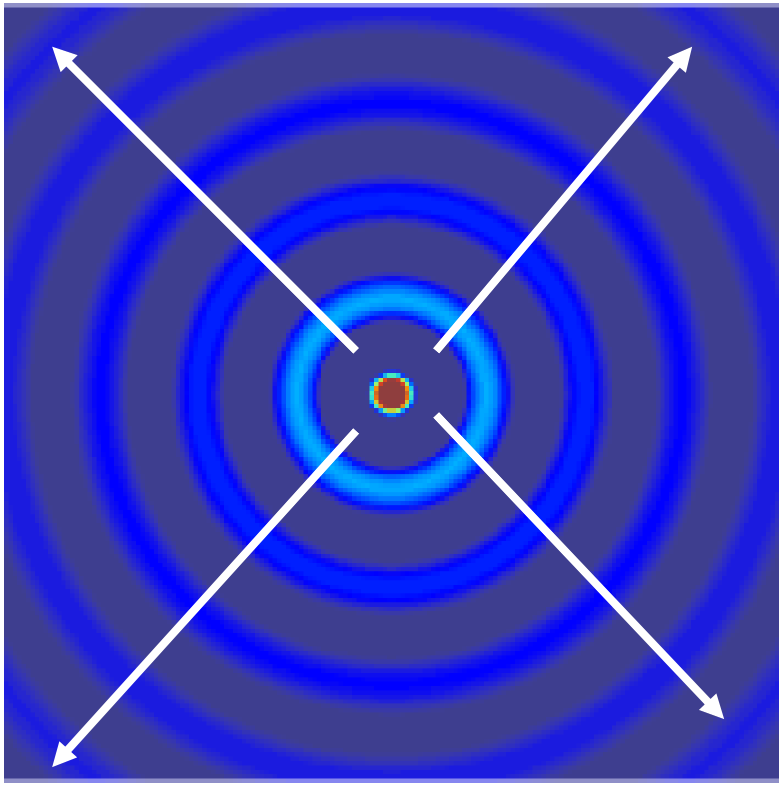




displacement at a single point

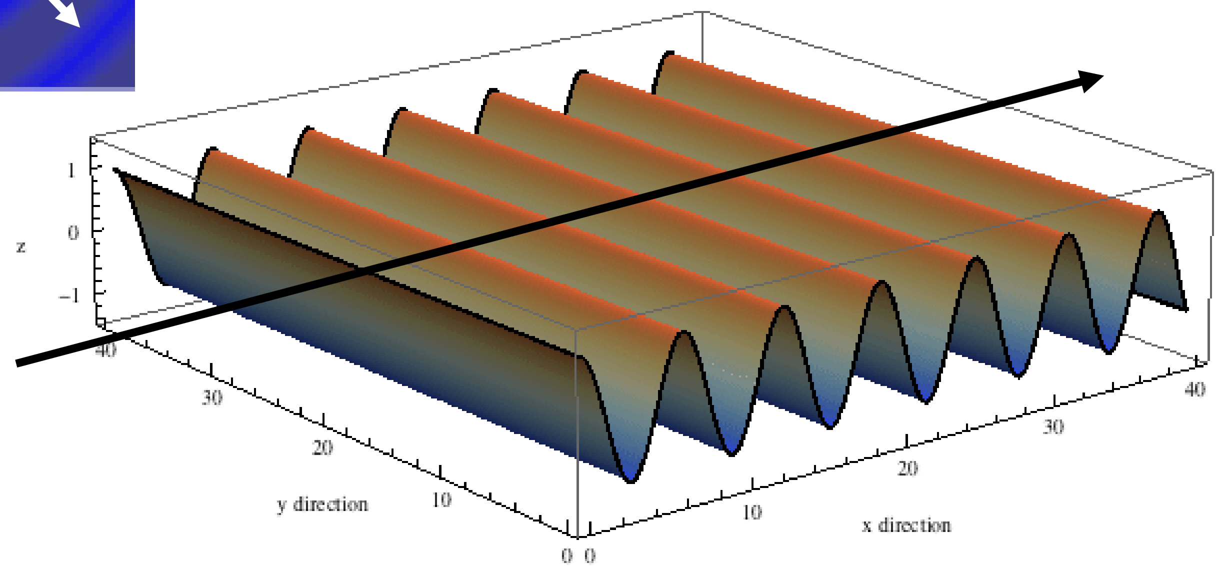


point-like source in a homogenious material
(waves propagate in all directions)



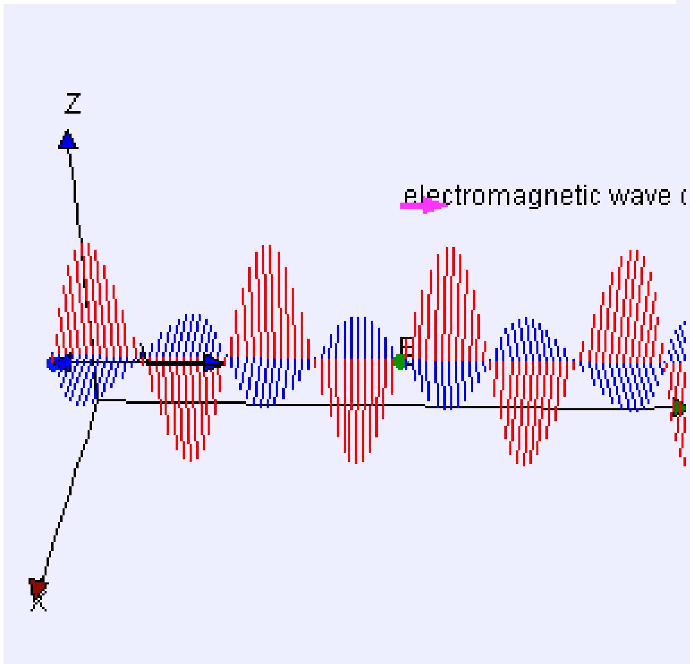
the propagation direction is everywhere perpendicular to the wavefront (or the curve connecting maximal displacement points)

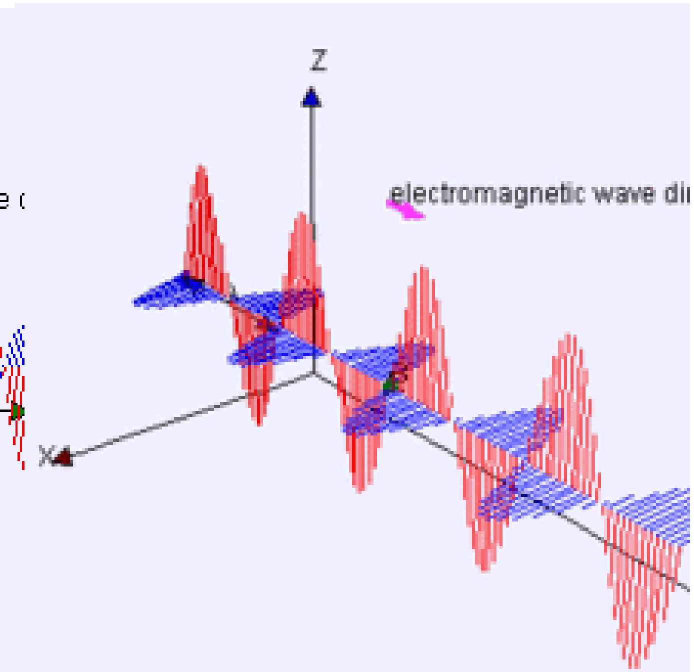
planar wave

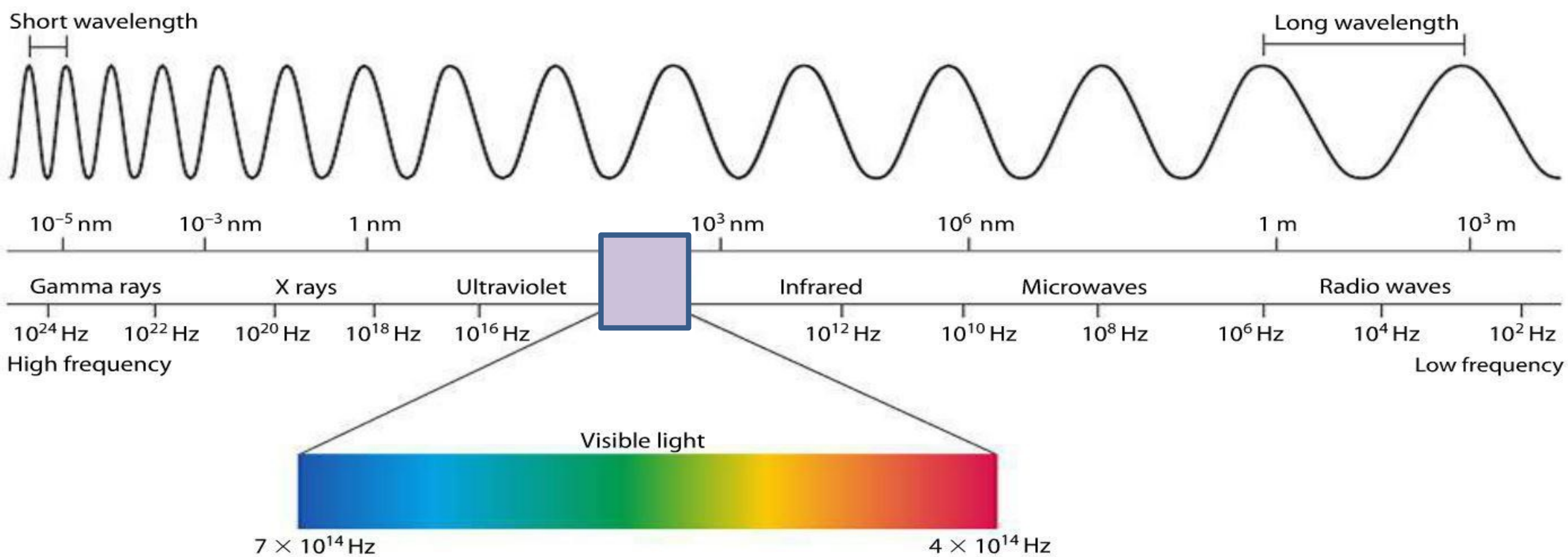


Light is an electro-magnetic wave

two waves entagled: electric field (**E**) and magnetic field (**B**) waves.

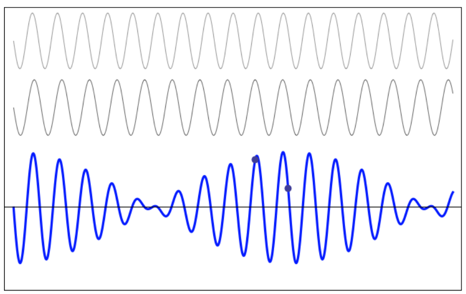


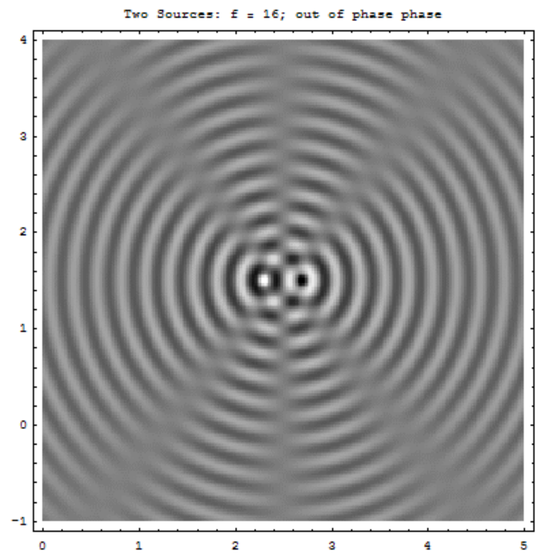


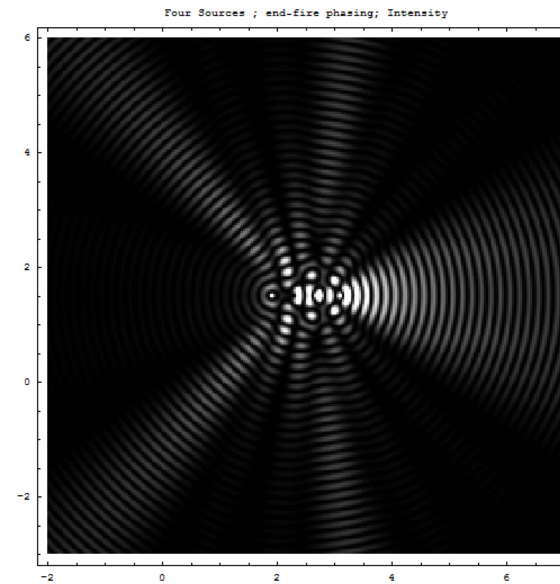


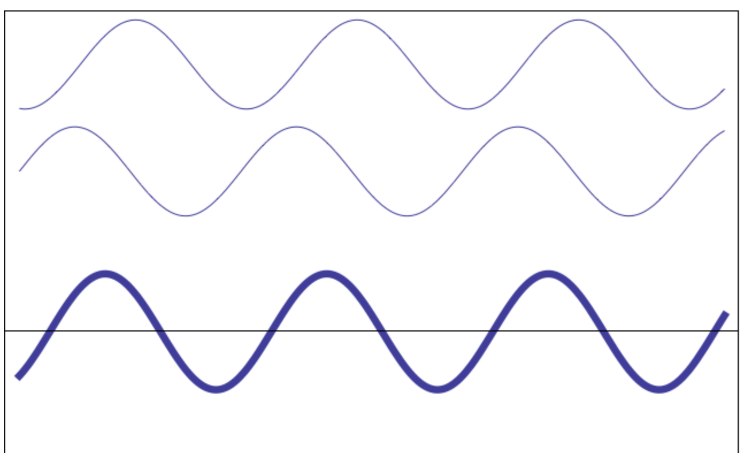
Superposition of waves:
the “displacements” caused by each individual waves add up at every point

$$y(x,t) = A_1 * \sin(k_1*x + \omega_1*t + \phi_1) + A_2 * \sin(k_2*x + \omega_2*t + \phi_2) + \ldots$$

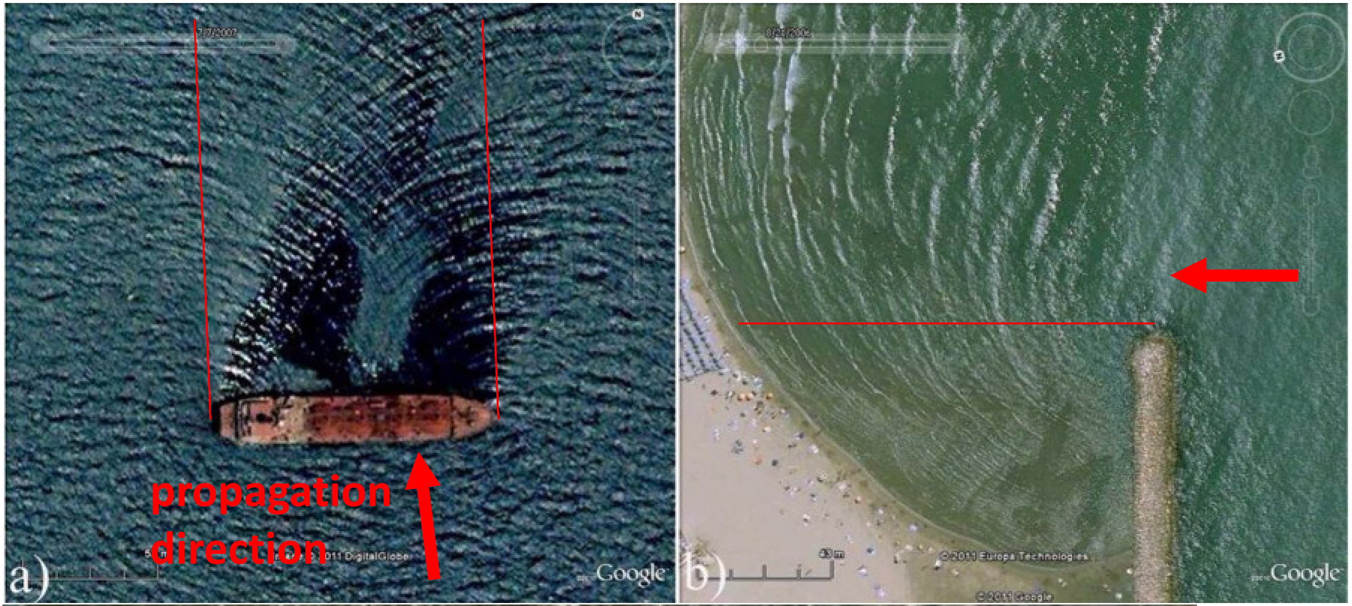


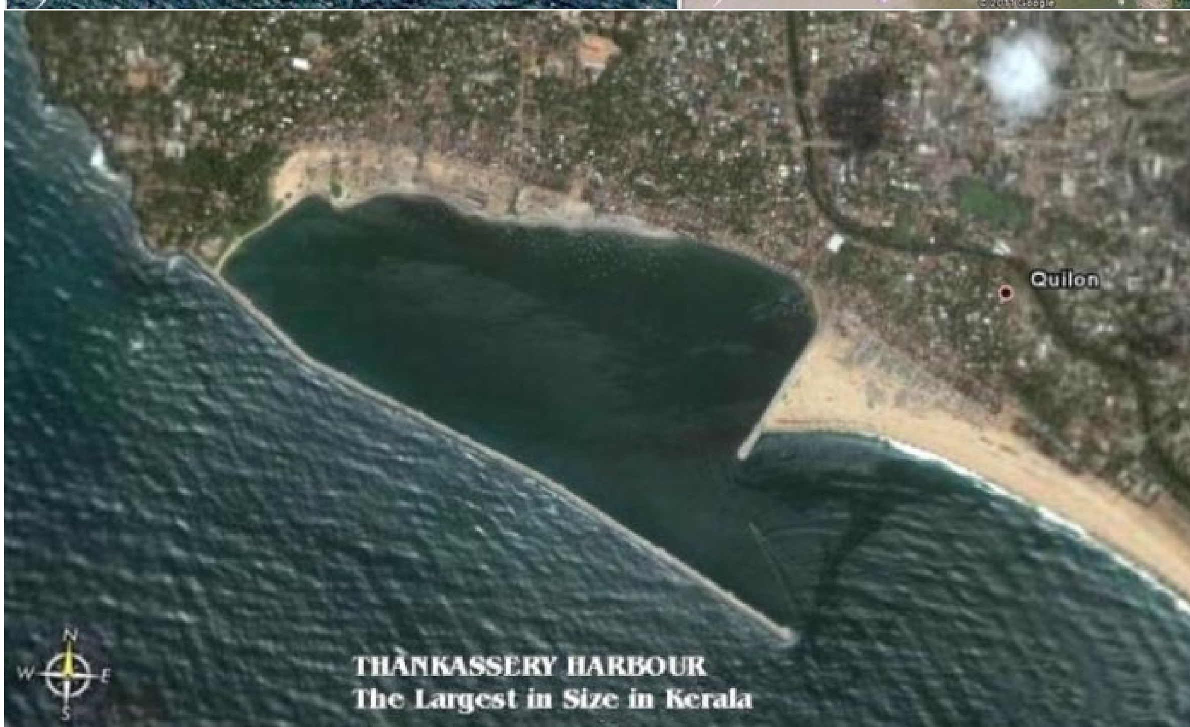






https://www.acs.psu.edu/drussell/demos/superposition/superposition.html

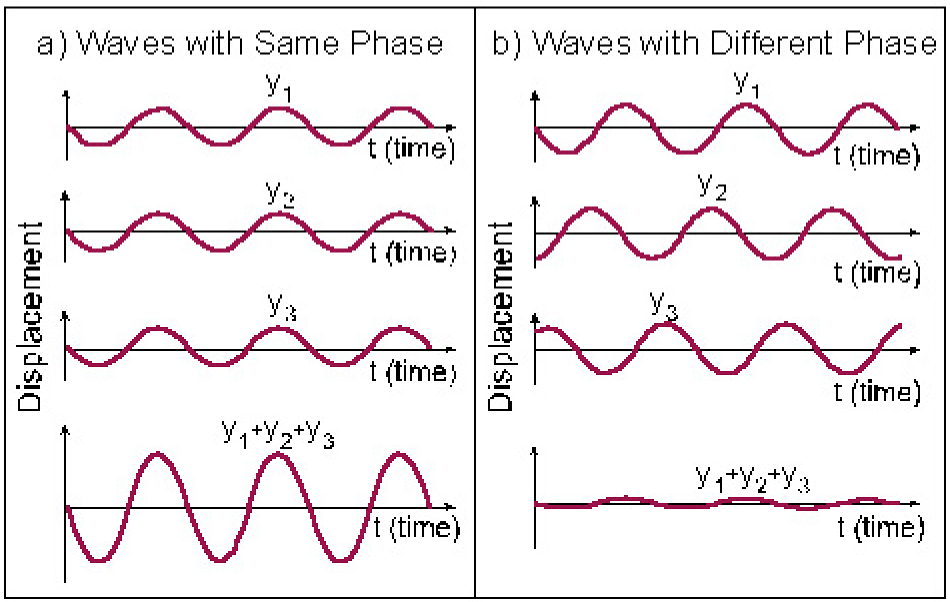


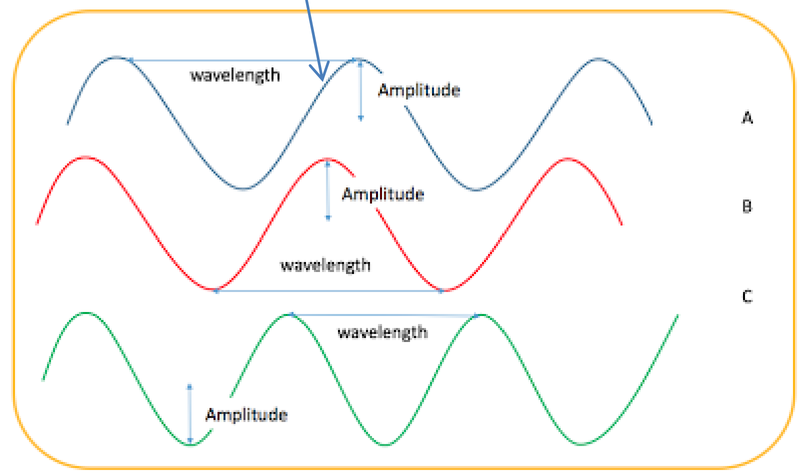


Diffraction of waves on water surface

Waves can be coherent or incoherent.
Coherent waves have the same frequency and a constant phase difference

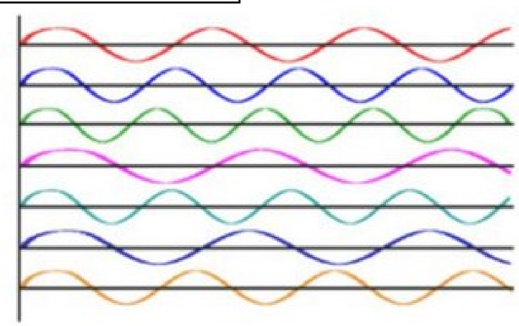
Only coherent waves can have a stable interference pattern, incoherent waves on average sum up close to 0.

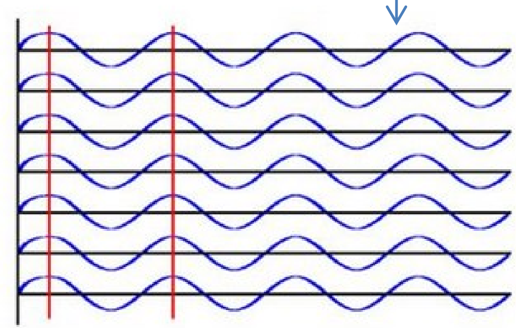




If the phase difference is 0 (or 1,2,3.. *2π), we have constructive interference.

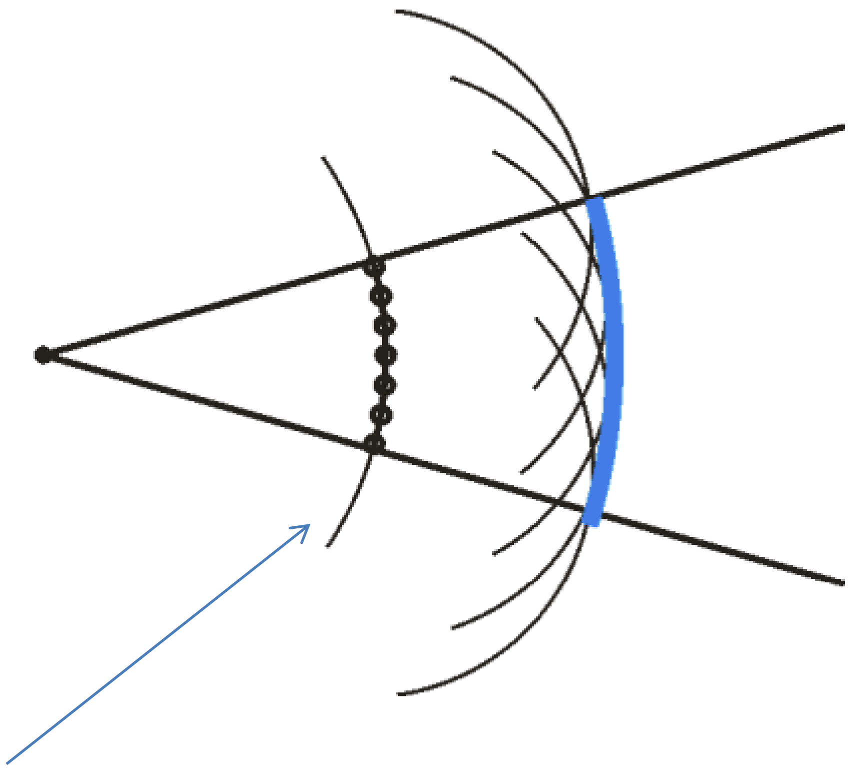
Constructive interference:
the resulting amplitude (A_{sum}) after summation is maximal.
Destructive: $A_{sum} = 0$

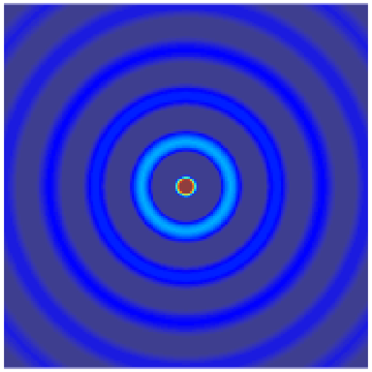

Incoherent light waves


Coherent light waves

The Huygens-Fresnel principle describes the propagation of waves, it can be used to explain most of the experimental results (but not the quantum-mechanical ones!)

In short: every point at the wavefront acts as a new sperical point source, and the resulting new wavefront can be computed as a superposition of all of the waves generated in this way.





Christiaan **Huygens** (1629-1695)

Augustin-Jean **Fresnel** (1788–1827)

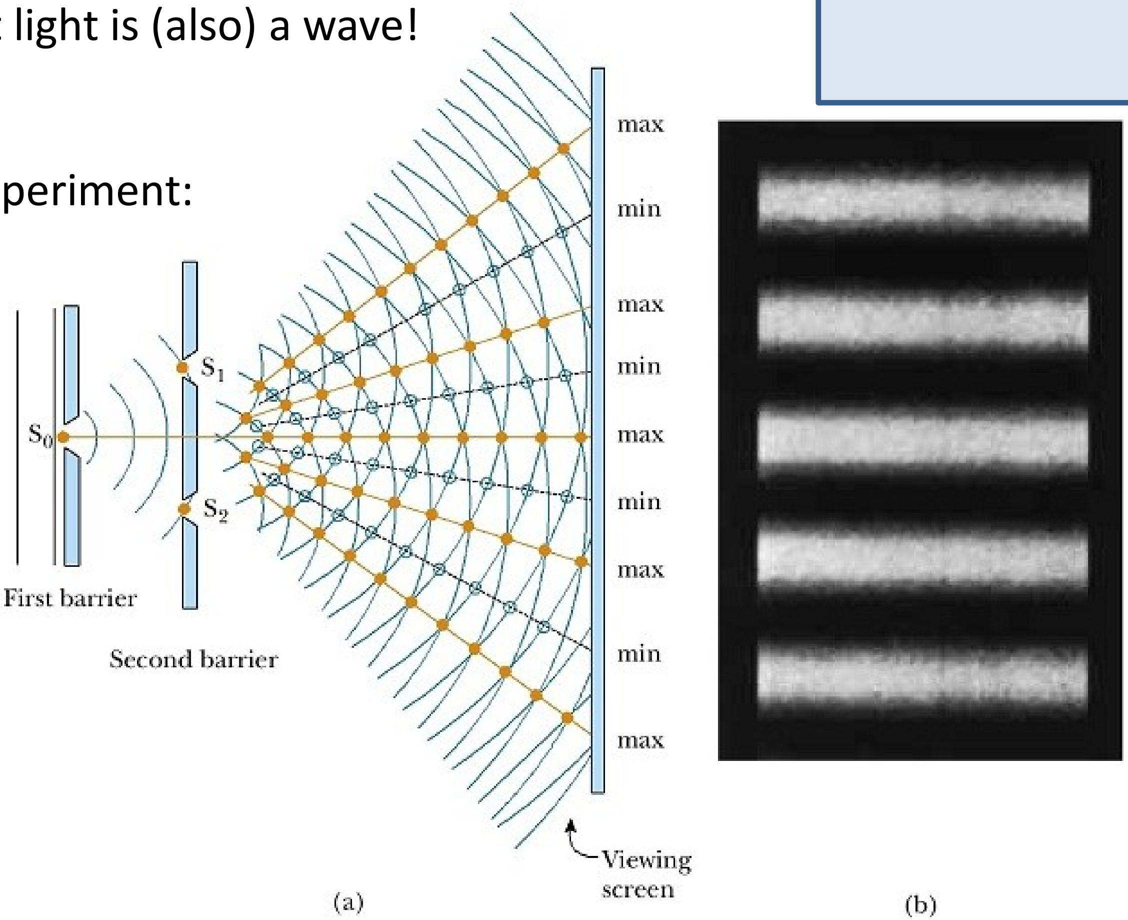
Wavefront: a surface containing points affected in the same way by a wave at a given time.

Some experiments, observations, which can only be understood with the help of wave theory.

Young’s two-slit experiment
Diffraction on grating
Interference patterns

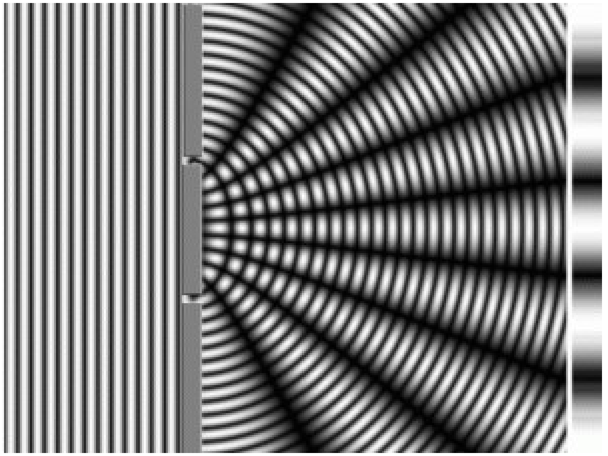
this proves that light is (also) a wave!

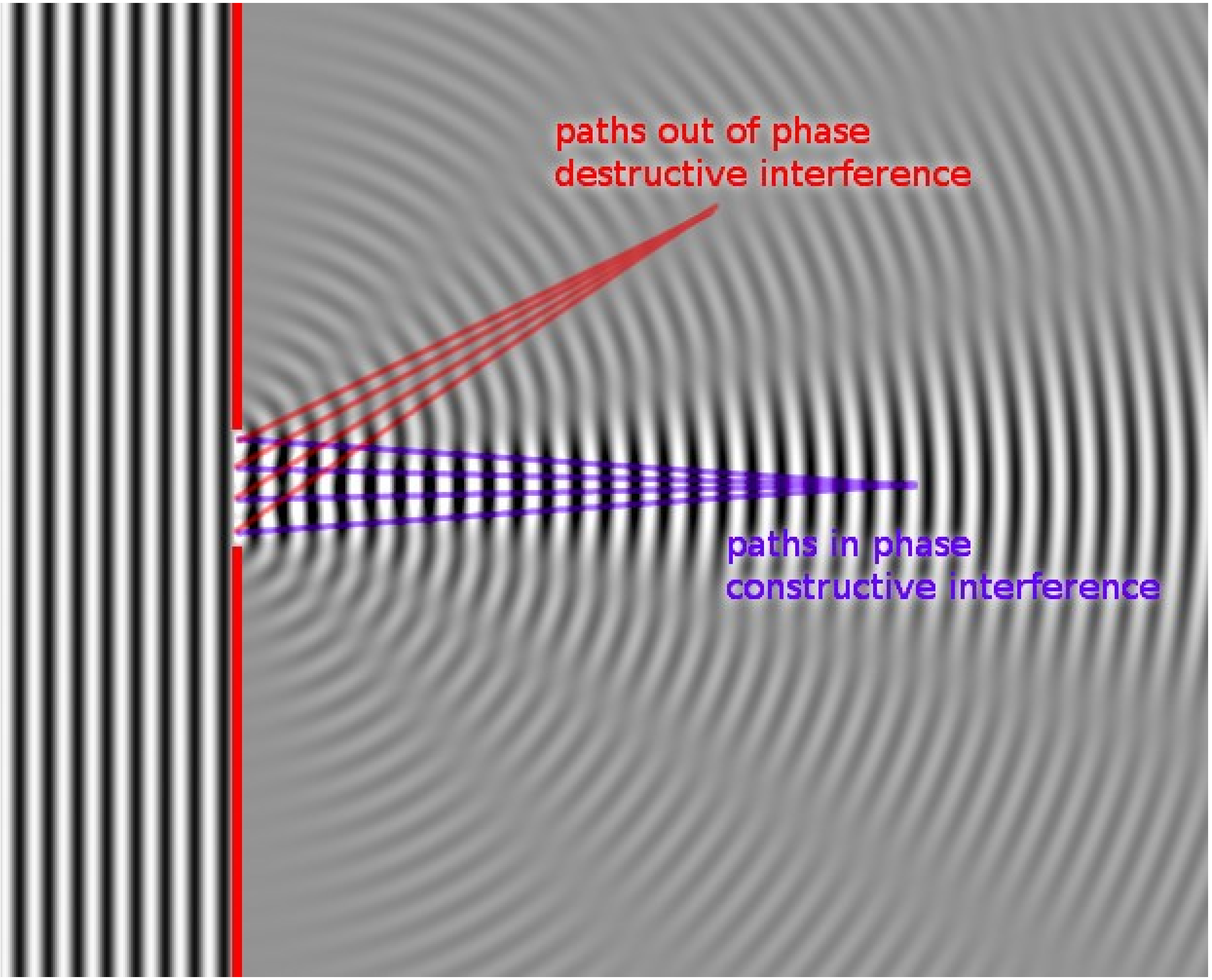
The two-slit experiment:



Thomas Young

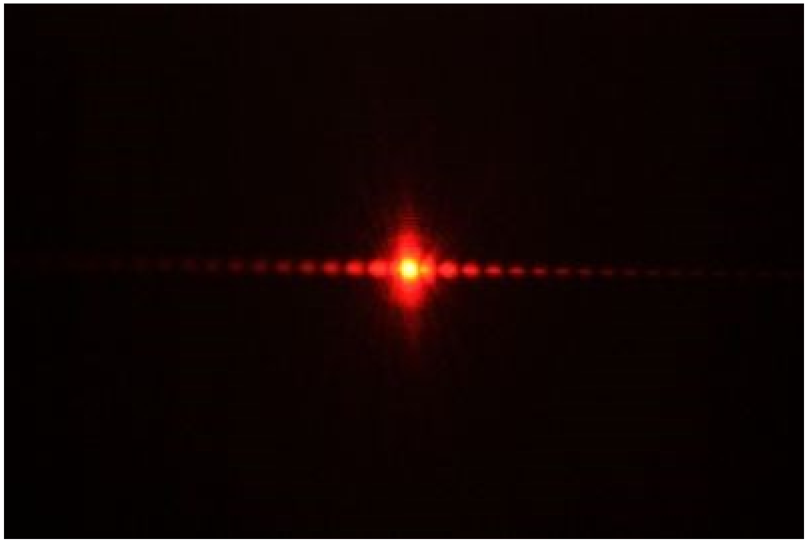
To understand the patterns we need to calculate the phase for each wave at the screen.

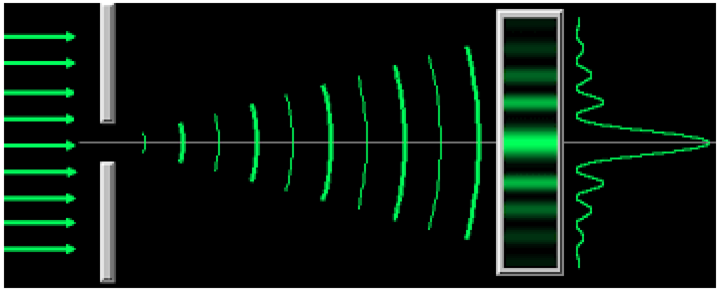


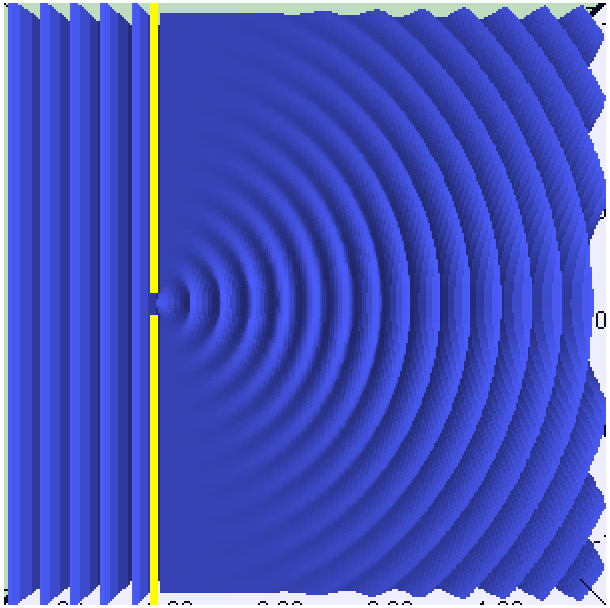


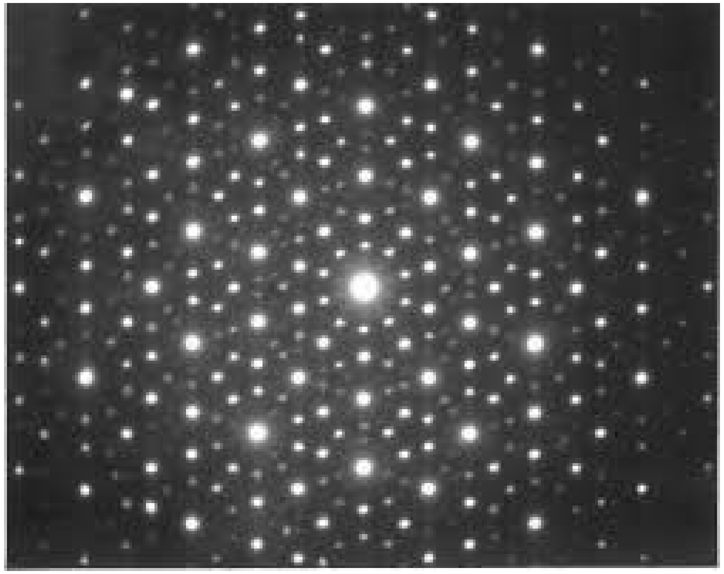
we get a noticeable pattern IF the size of the slit or slit-distance is comparable to the wavelength.

Diffraction and interference patterns with coherent light





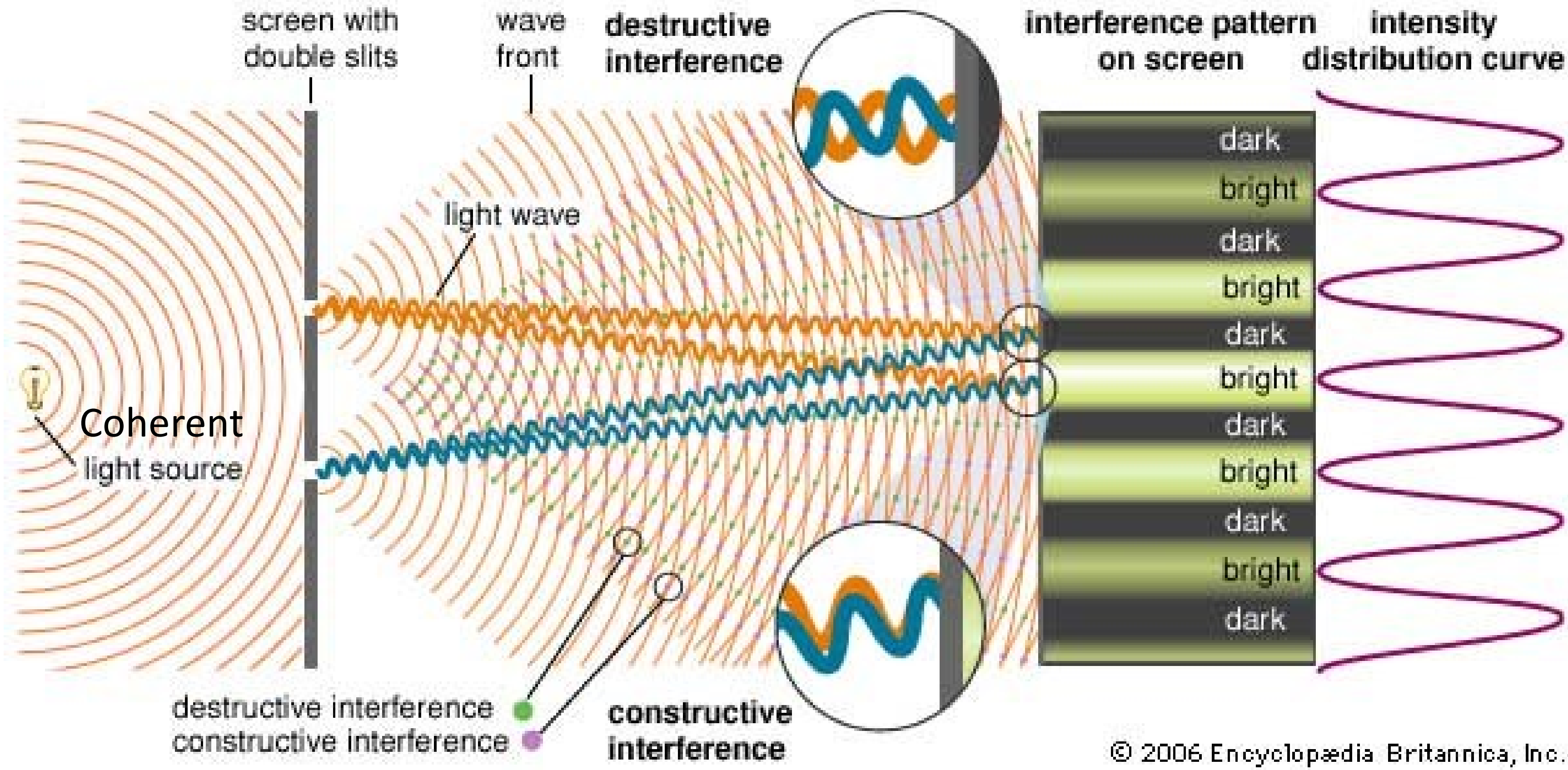


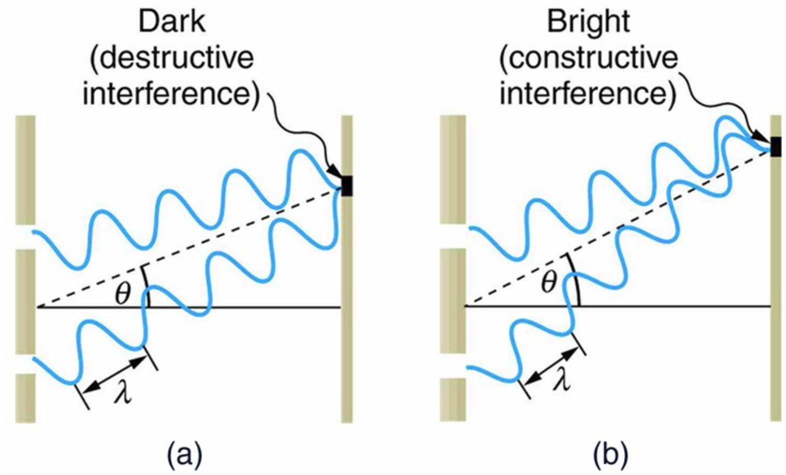


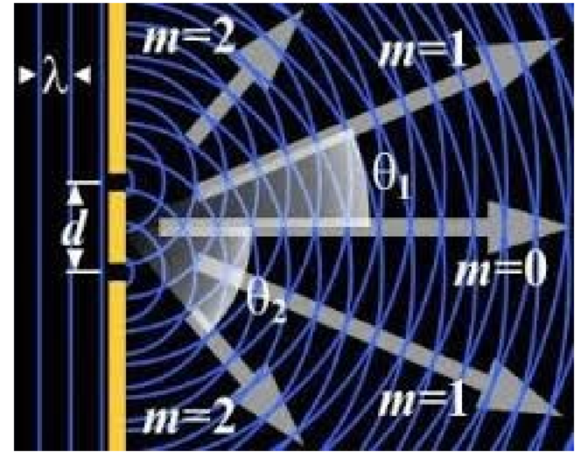
for very small grids -> crystals

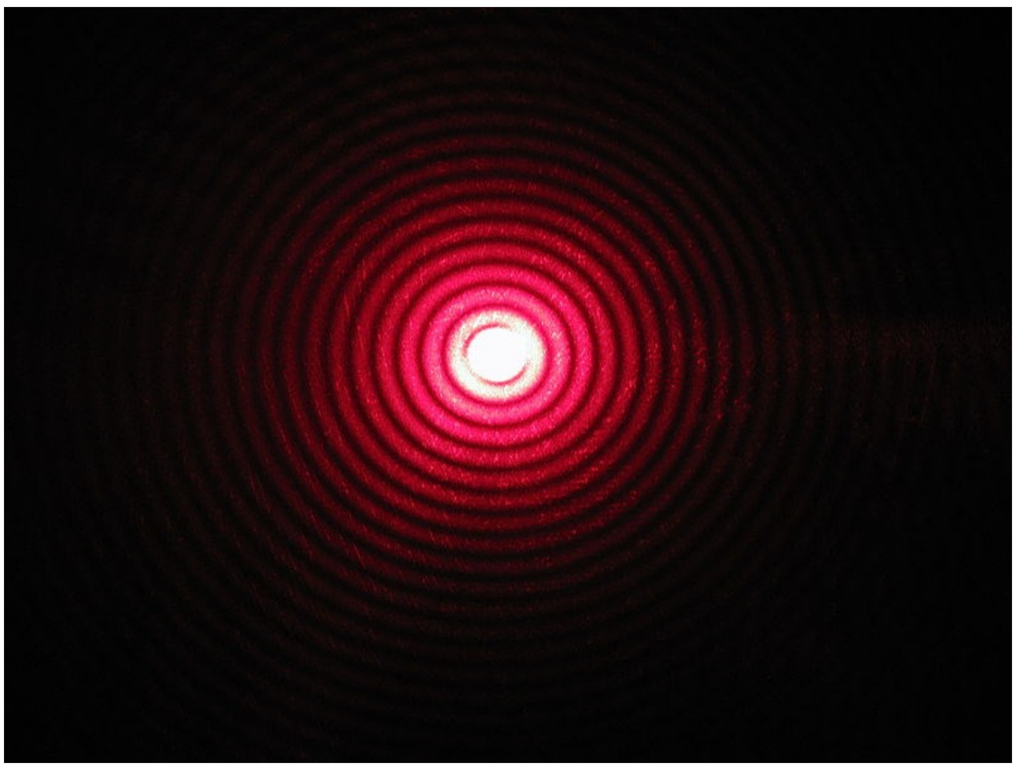
we need short wavelength -> pm -> X-Ray light

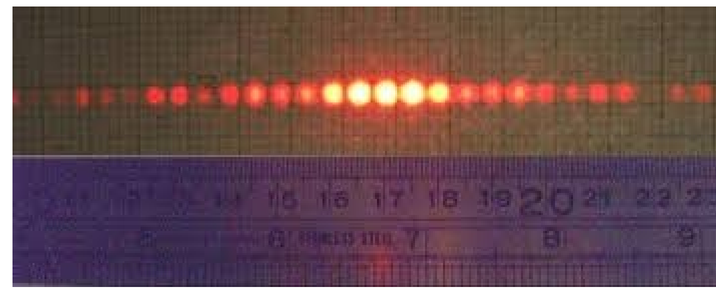
The explanation of the periodic profile : interference of waves, Huygens principle.




Dark
(destructive
interference)
Bright
(constructive
interference)
θ
λ
(a)
(b)


λ
d
m=2
m=1
θ1
m=0
θ2
m=1
m=2

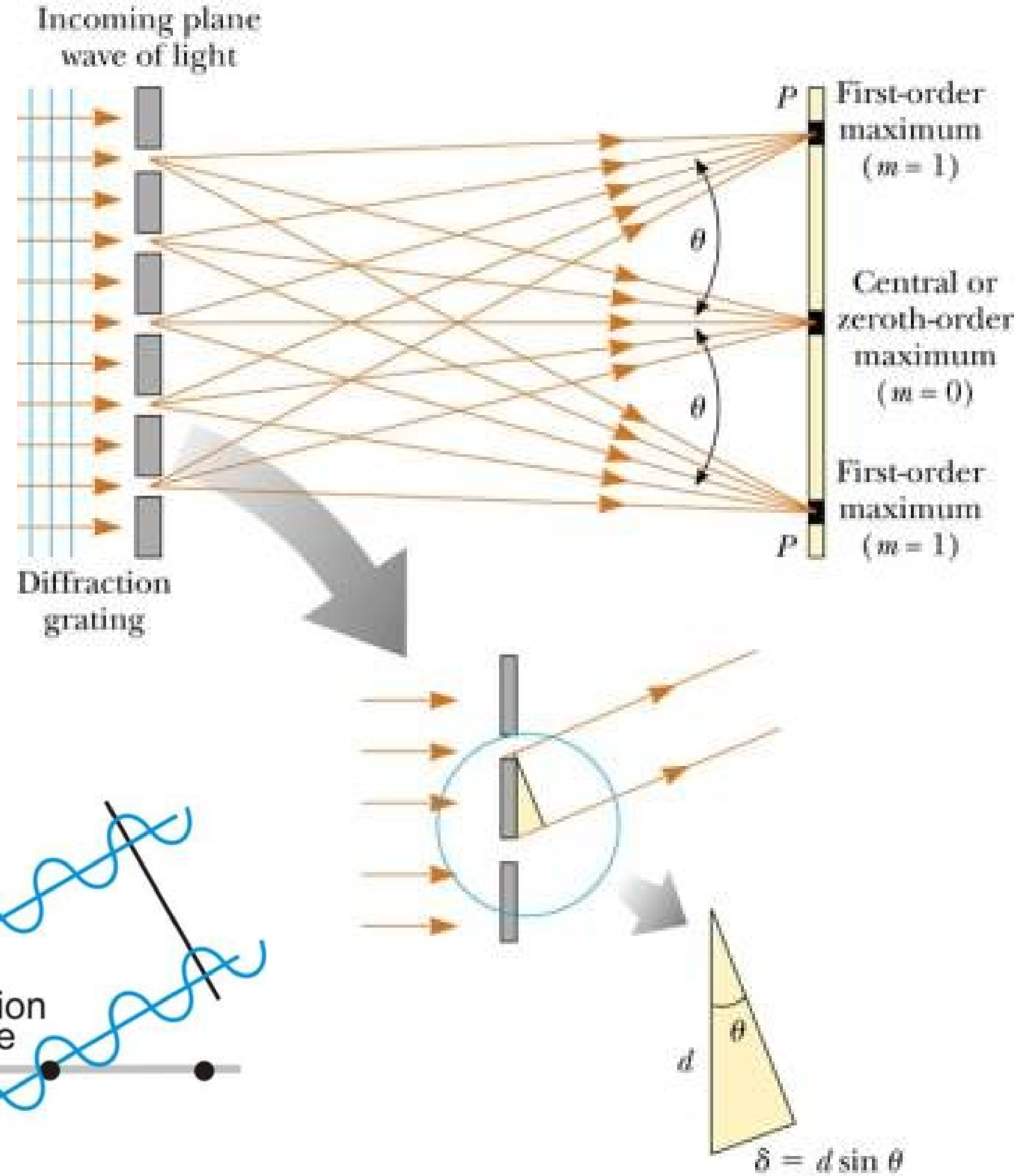




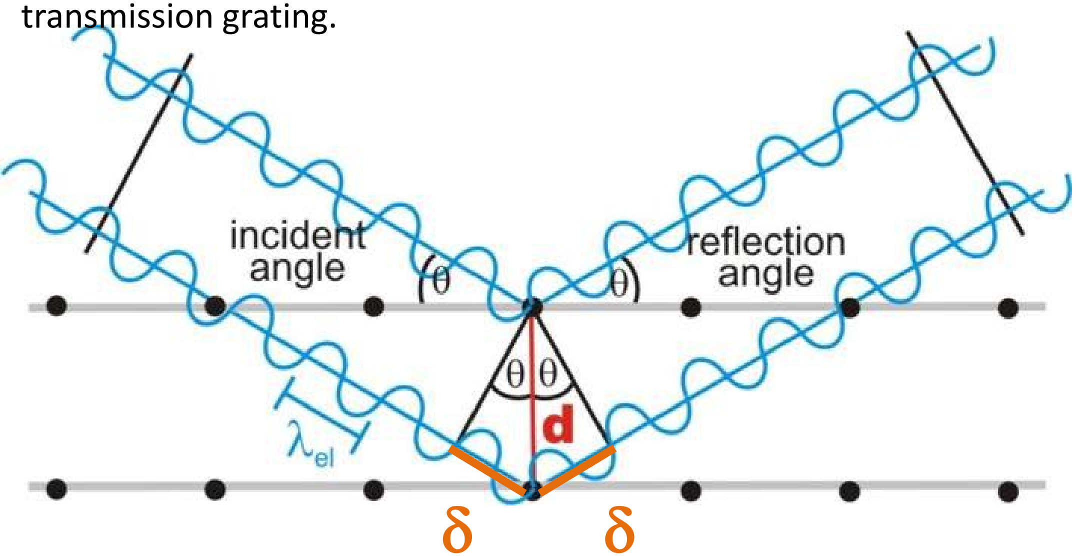
Diffraction on optical grating:

Due to the path difference of δ a phase difference of $2\pi * \delta/\lambda$ arises between waves.

(n.B.: during a length of λ, the time it takes for the light to travel is T,
under which the change in the phase is exactly 2π)



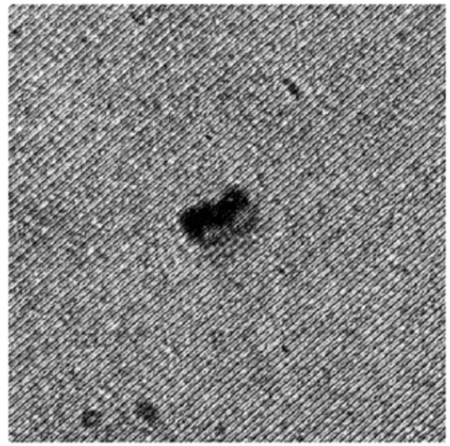
On a *reflection grating* the concept is the same, but the phase difference (due to **path difference 2*δ**) is *twice* as much as on a transmission grating.

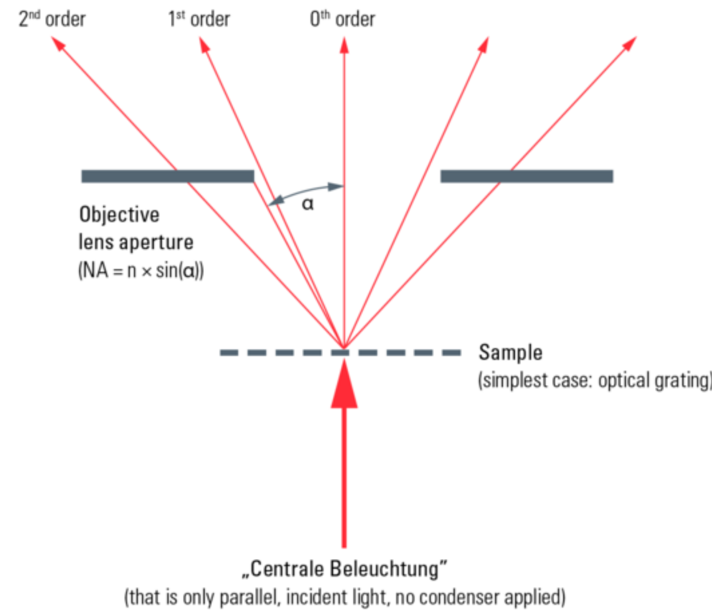


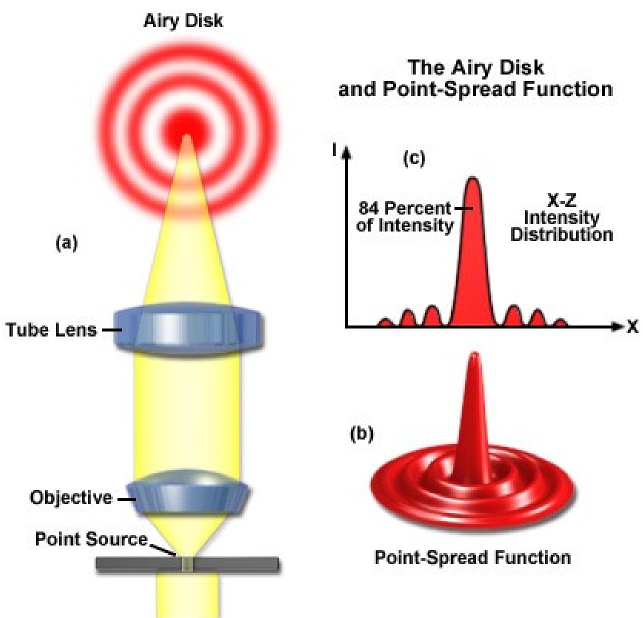
Constructive interference is only possible if the phase difference is 0,1,2,3,... * 2π. This means that δ has to be 0,1,2,3,... * λ

Resolving power of a microscope is limited by the wave's diffraction.

Abbe's principle:
There is **only** an image formation in the microscope **if** at least the first order diffracted waves enter the objective lens.



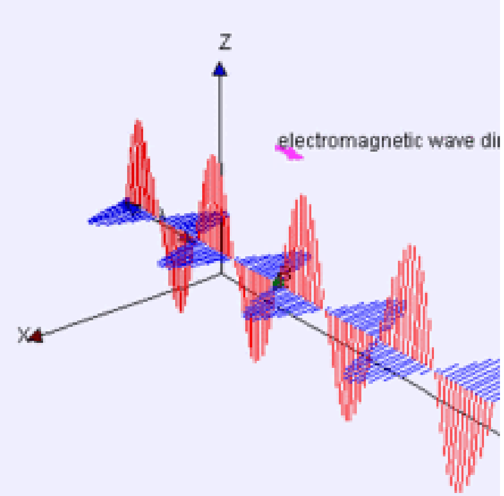




$$d = \frac{\lambda}{2n\sin\alpha}$$

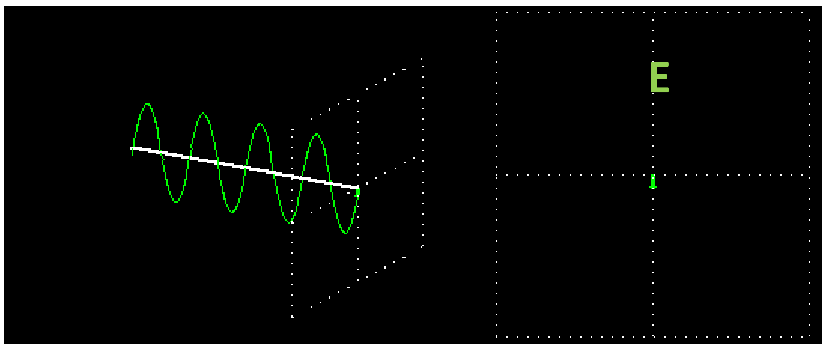
Ernst Abbe
(1840-1905)

modern version: $\delta = 0.61\dfrac{\lambda}{n\sin\omega}$



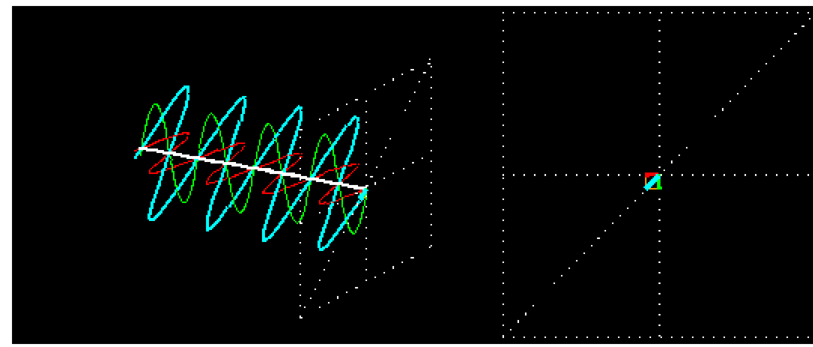
Polarization

polarization direction: the direction of the *electric filed*'s vector in the wave

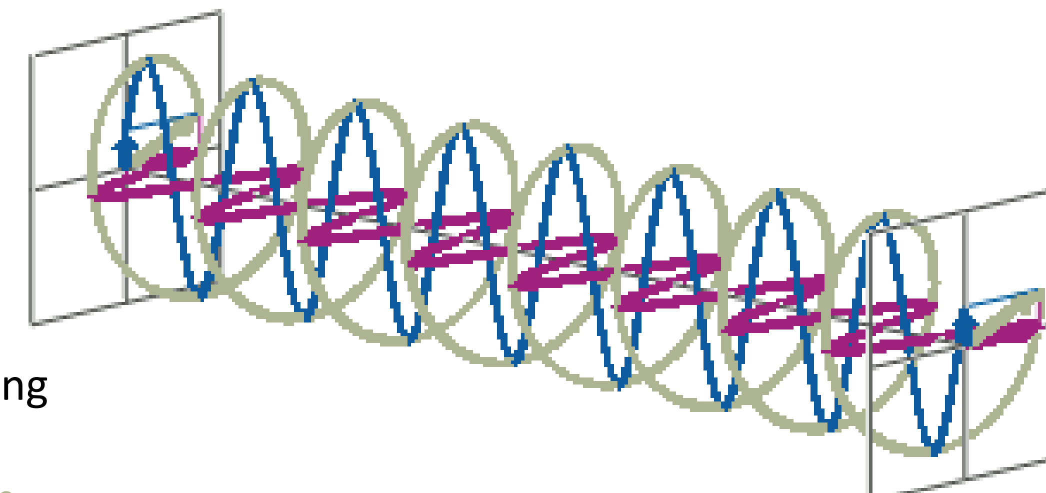


linearly polarized light

Summation of two waves with different E field direction, but same phase



circularly polarized light

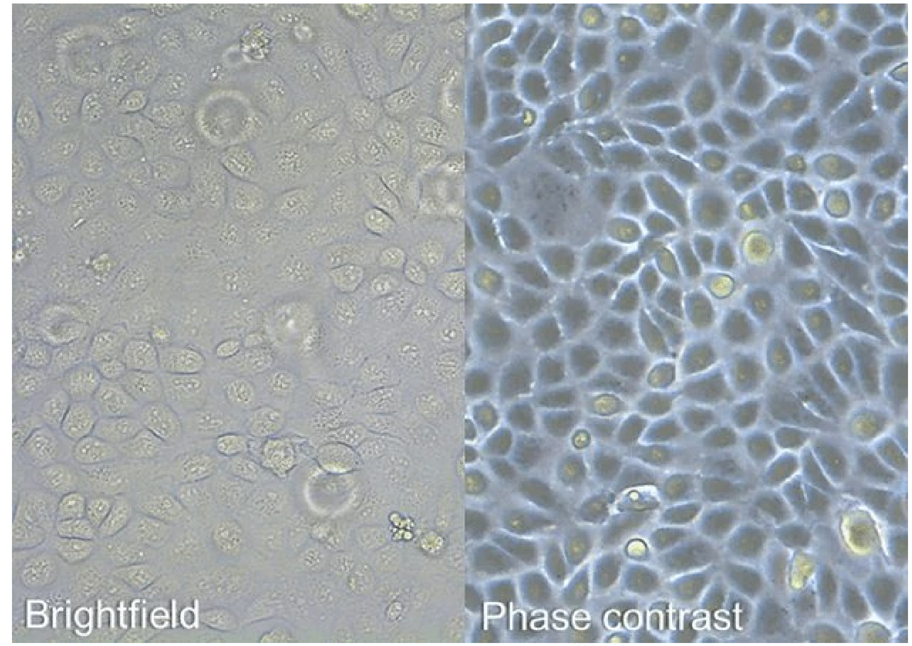


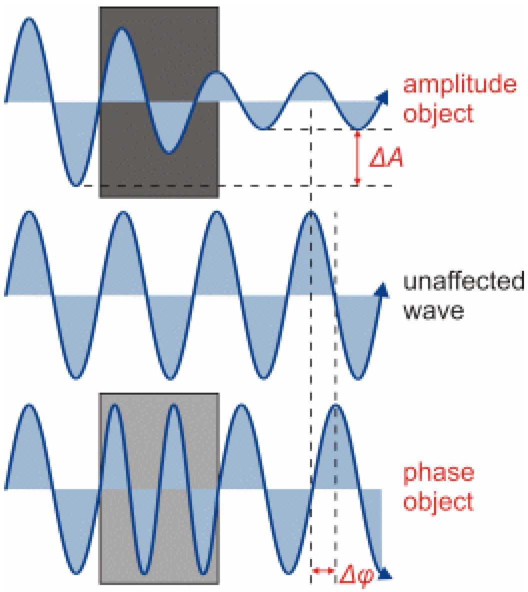
it is essentially a sum of two waves having 90 deg. phase difference

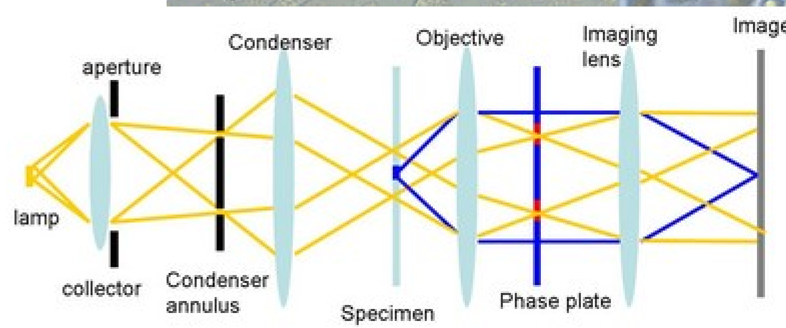
vertical + horizontal = rotating sum

Phase contrast microscopy

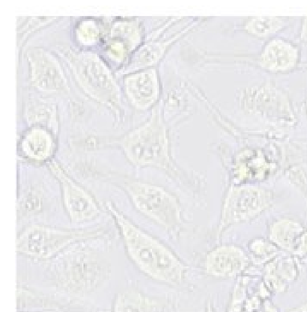
The phase object is transparent,
but the refractive index is different,
so the speed of light is different,
which builds up a phase difference during
the way through the object.
This is not observable to the human eye, but
can be made visible with the phase-contrast
microscope, which creates intensity changes
from the phase changes.

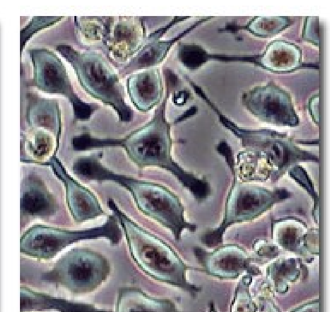






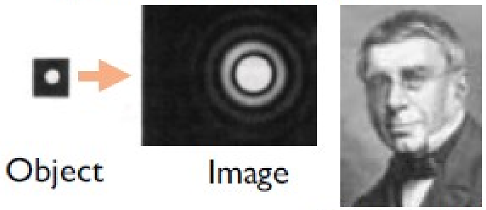
It creates the contrast (amplitude change) by coherently re-interfering a reference, or surround beam (S), with a diffracted beam (D) from the specimen. The S beam is shown in yellow, and the D beam is shown in blue.





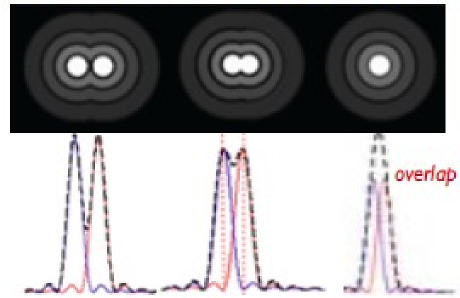
Diffraction is also present in the human eye:

Because of diffraction: image of a point object is an Airy disk



Sir George Biddel Airy (1801-1892)

Rayleigh criterion: objects may be resolved if their corresponding Airy disks do not overlap



Lord Rayleigh (1842-1919)

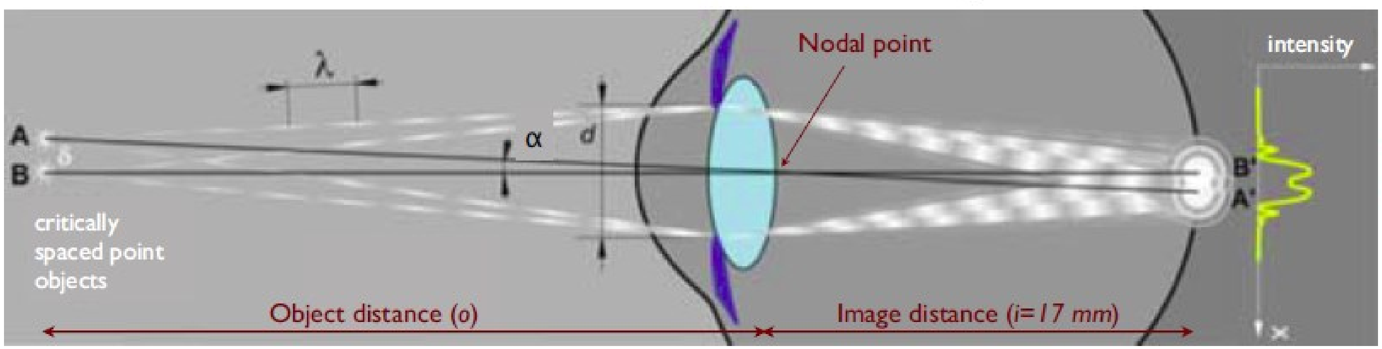
Smallest resolved distance has a limit (Abbe equation):

$$d = \frac{0.61\lambda}{n \sin\alpha}$$

λ = wavelength
n = refractive index of medium
α = angle between axis and outermost ray

Emst Abbe (1840-1905)

Diffraction limit of the human eye

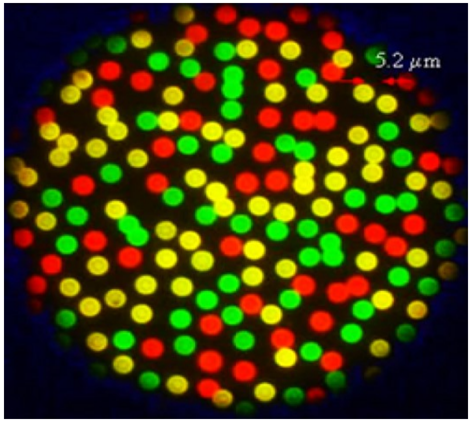


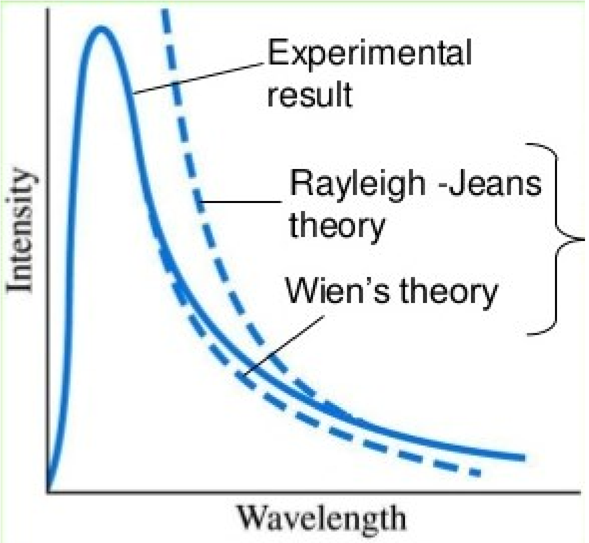
Reduced

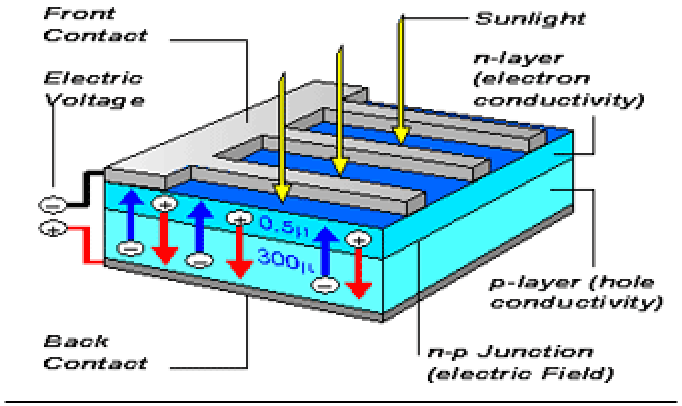
For a detailed explanation, and more figures: see the “optics of the eye” lab manual!

There are some experiments, observations which still can not be explained alone by the wave theory

Photoelectric effect
Quantum dots
Fluorescence
Lasers
Black body radiation







Back contact solar cell (Courtesy: ECN, The Netherlands)

-> **light is a wave AND a particle (this leads us to quantum physics)**

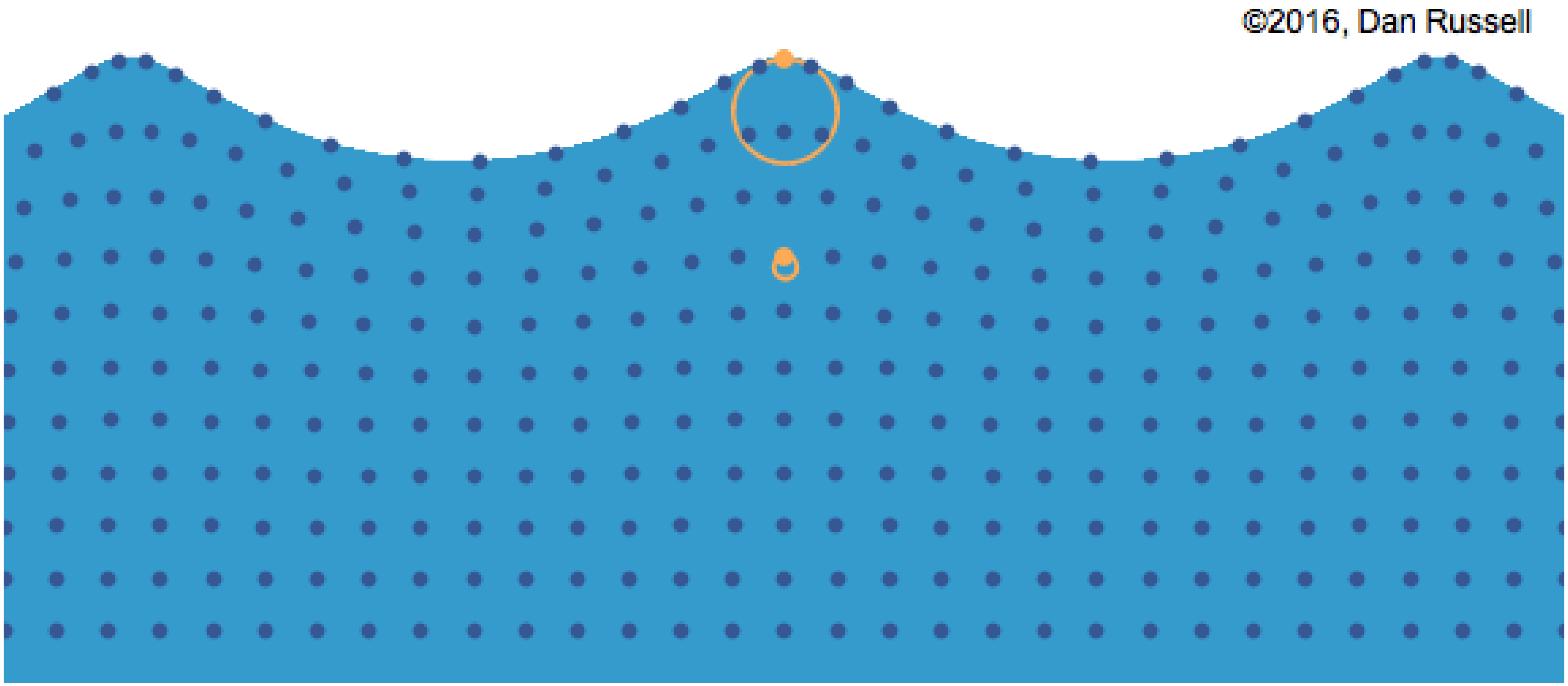
the energy is transported in "packages", quanta: $E=N \cdot \varepsilon$, $\boldsymbol{\varepsilon = h \cdot f}$

Planck's constant $h = 6.6 \cdot 10^{-34}$ Js

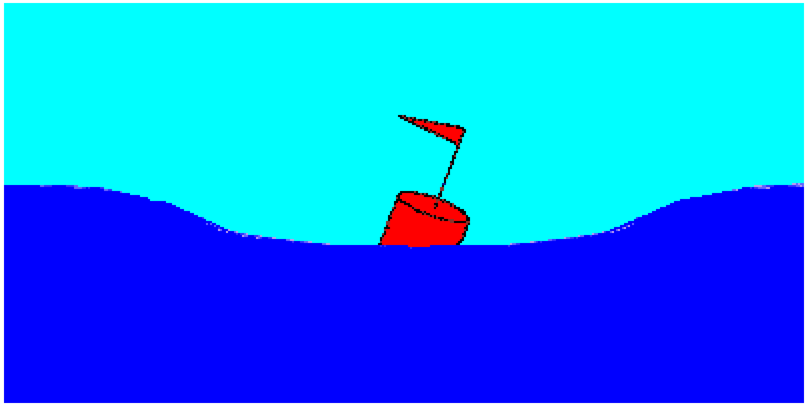
END of lecture material

The next slides contain additional learning material, but are not part of the reduced, 45-min lecture.

Waves are seen most often in water:

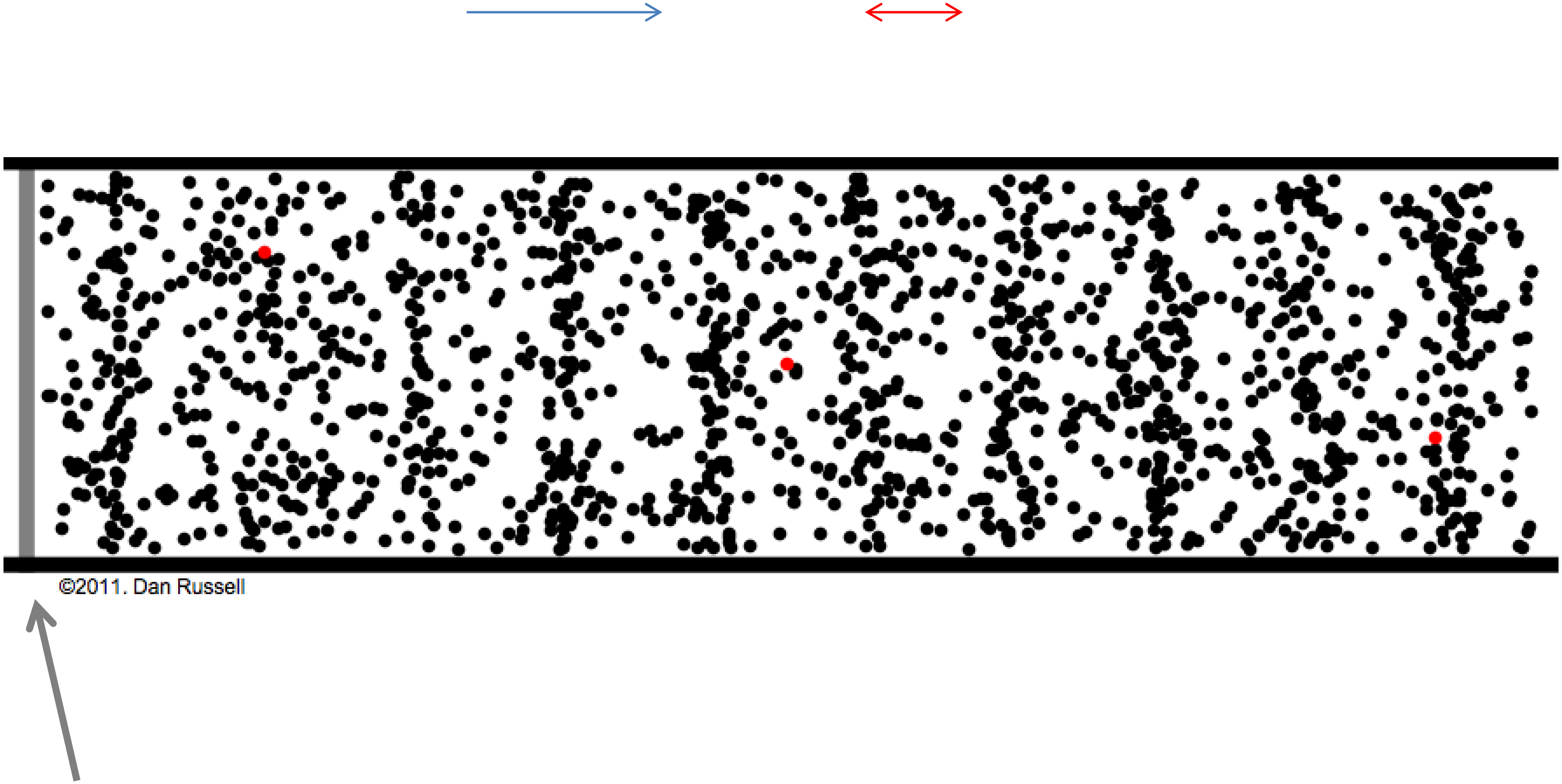


Waves can be described by the wave equation, which relates
the motion of individual parts of the medium to the observed wave.



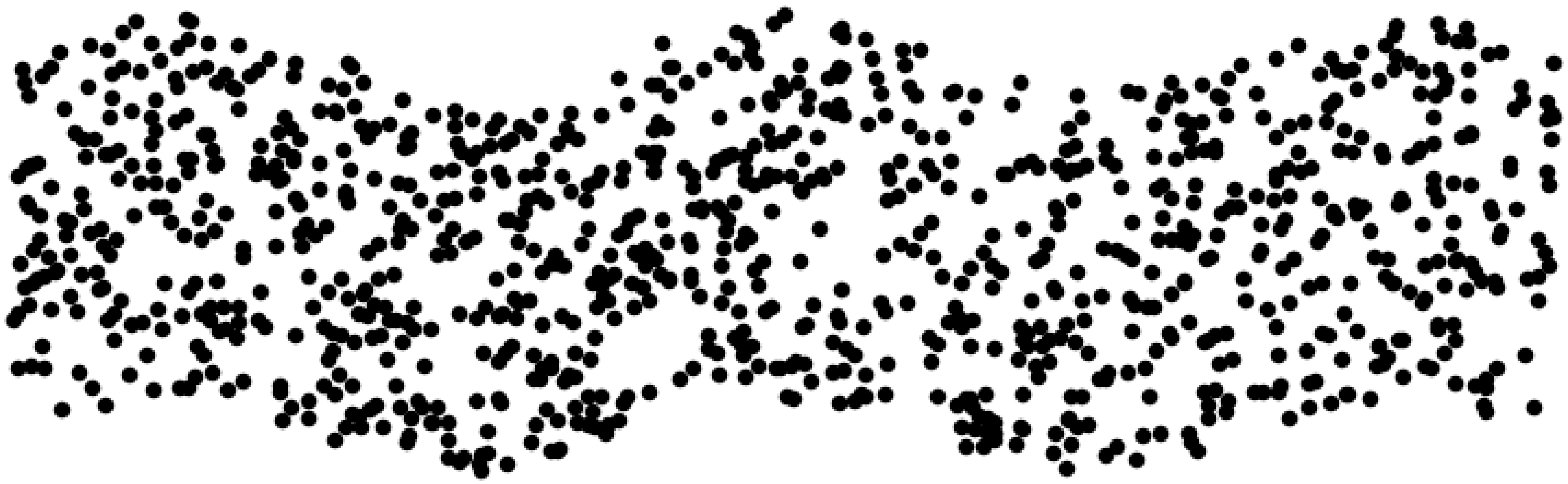
It is important to note, that as the waves propagate, the parts
of the medium (here the water molecules) stay “in place”, which means
there is no net transport of material.

Longitudinal waves:
propagation direction is parallel to the “motion”



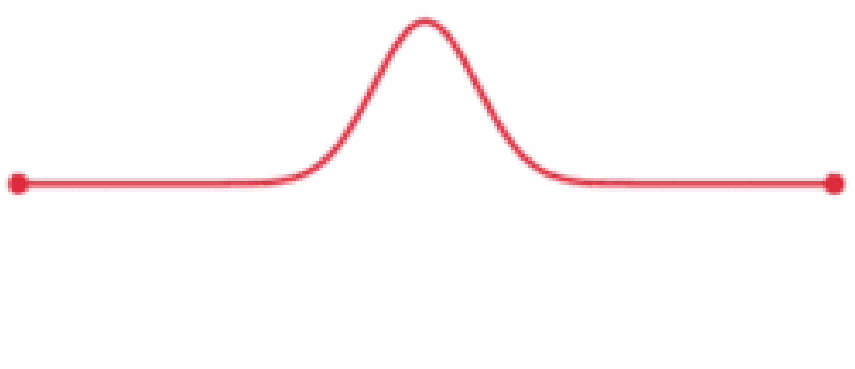
Moving surface (wave “source”)

Transversal wave – such as light, or sound in some cases in solids



Transversal: wave propagation is perpendicular to the “motion”

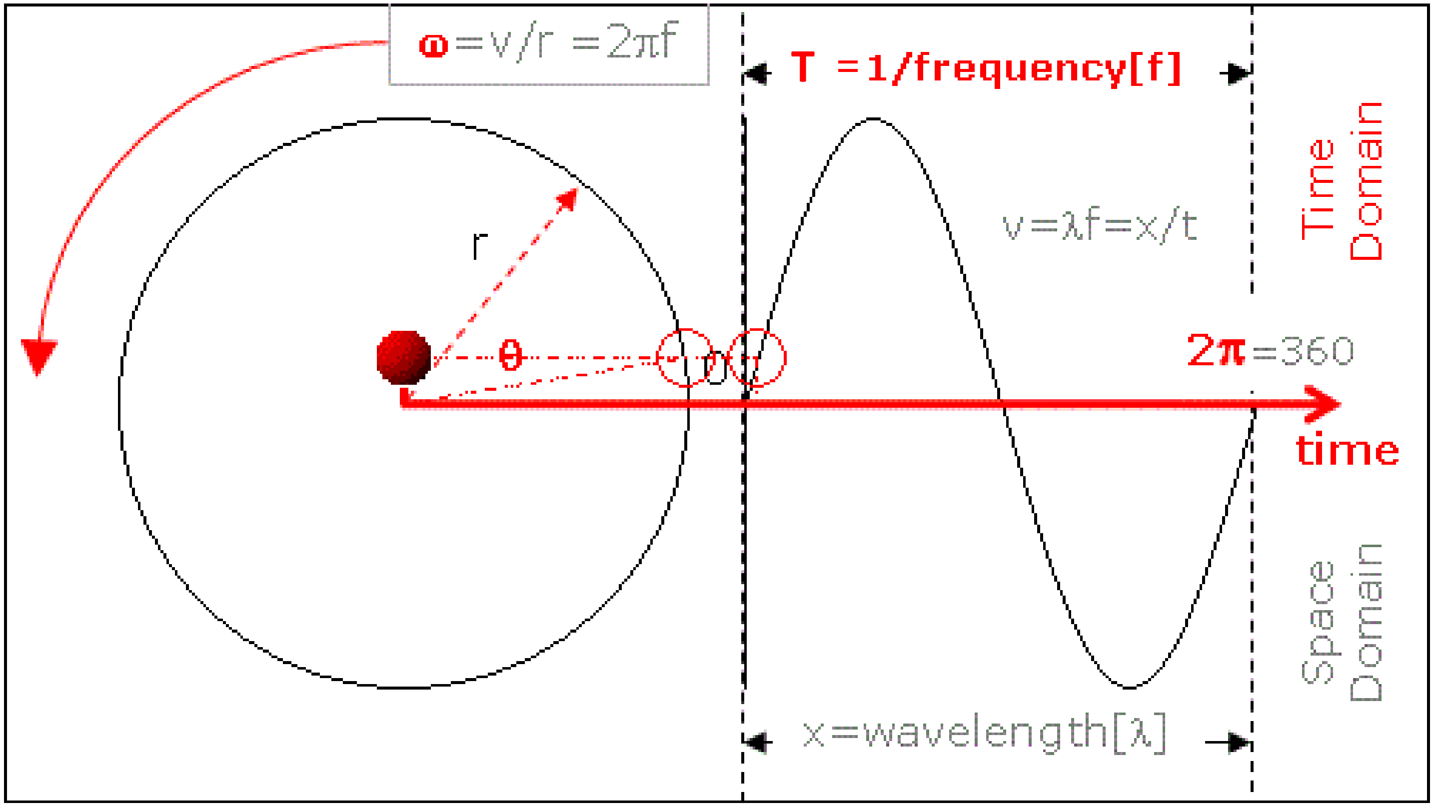
As an observation we can say that as waves travel or **propagate** the change in the state of the parts of the medium is moving.



Here the “state of the parts” simply means the deflection of the string at a given point.

The wave is moving forward (propagating) with a speed of "v", any point's position is dependent on both space and time.

Most important motion type is the harmonic motion. (produced by a harmonic oscillator)



The wave equation is a bit complicated:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

We take the change of any property (here “u”) in time (du/dt) and also in space (du/dx), but we need to take the change of the change (d^2u/du^2), and these are linked by the propagation speed (or other named phase velocity) (here as “c”).

A simple solution for u(x,t) is:

$u(x,t) = A * \sin(k*x + \omega*t + \phi)$

where

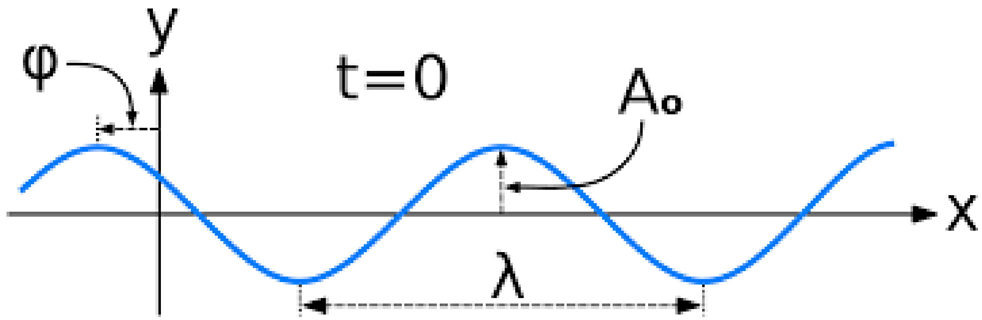
A is the amplitude of the wave, k is the wavenumber, and ω is the angular frequency

$\omega = 2\pi f$, where f = 1/T [Hz], while T is the period time.

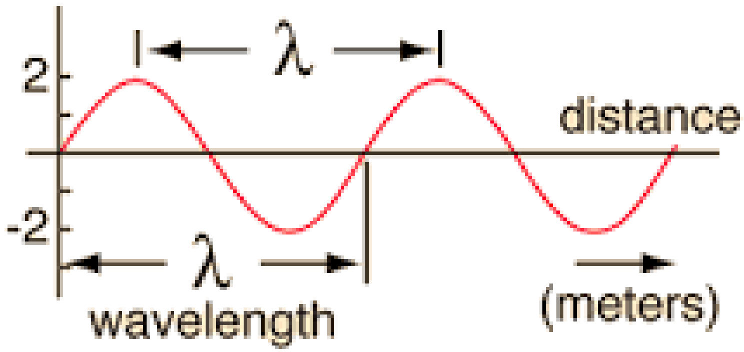
$\omega = c*k$ defines the wavenumber, which can be written as $k = 2\pi/\lambda$.

here λ is the wavelength.

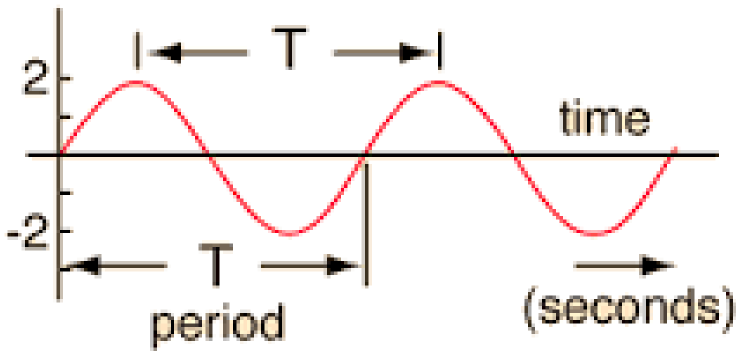
Graphical representation of the solution



Snapshot at a given time



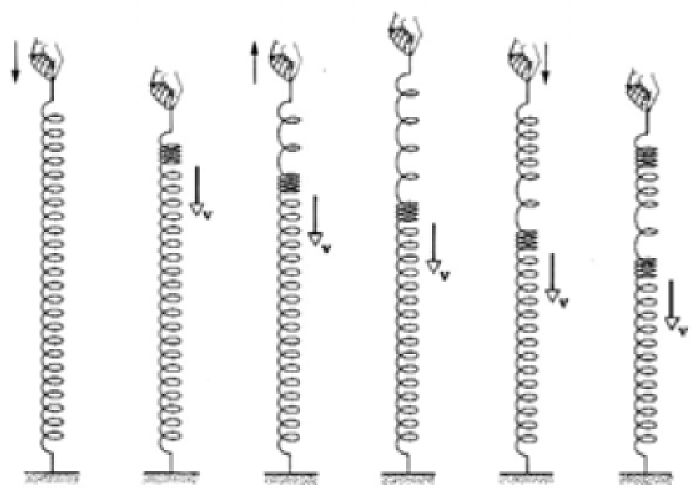
Time evolution at a given point



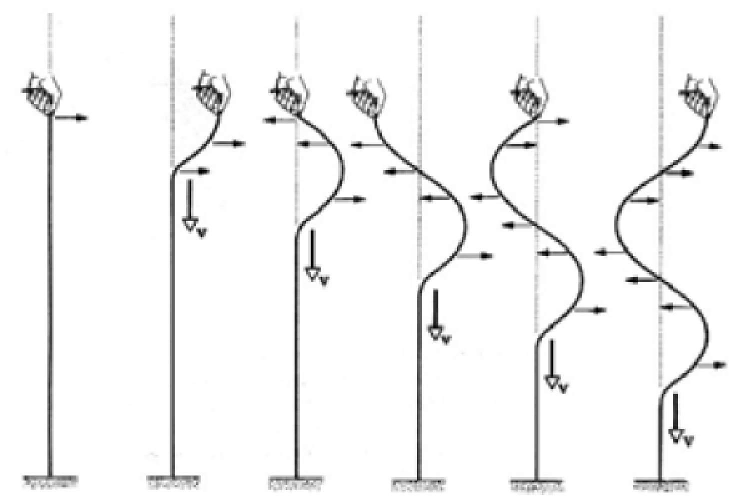
Different types of waves

- According to source:
  1. Mechanical: elastic deformation propagating through elastic medium
  2. Electromagnetic: electric disturbance propagating through space (vacuum)
- According to propagation dimension:
  1. One-dimensional (rope)
  2. Surface waves (pond)
  3. Spatial waves (sound)
- According to relative direction of oscillation and propagation:

1. Longitudinal



2. Transverse



For EM waves we have two equations, and the wave can travel to x,y,z directions, so the equations are a bit even more complicated:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$
$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

Here the ∇ means the $d^2/d\ldots^2$ in all directions

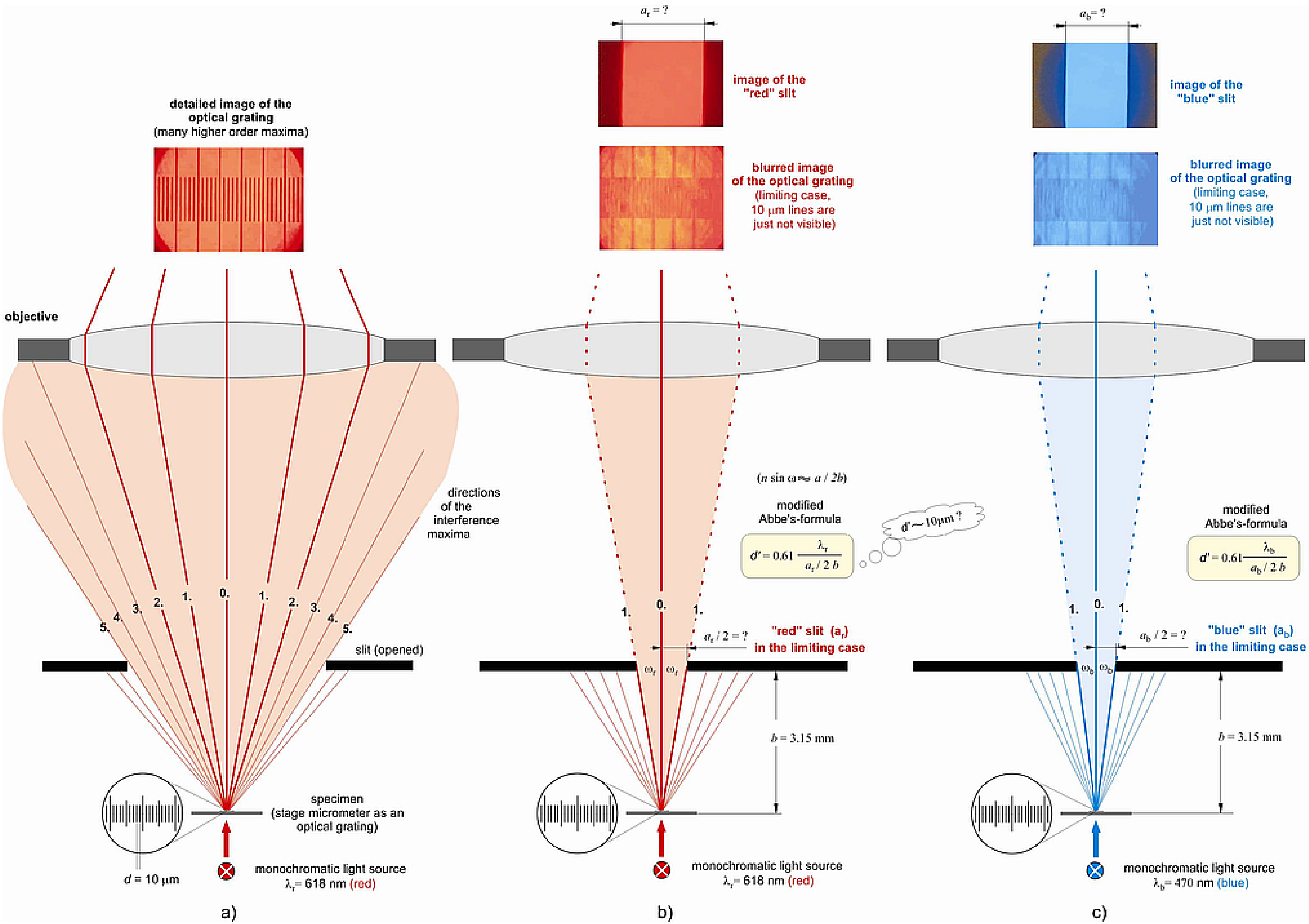
The solution is again a sine or cosine wave:

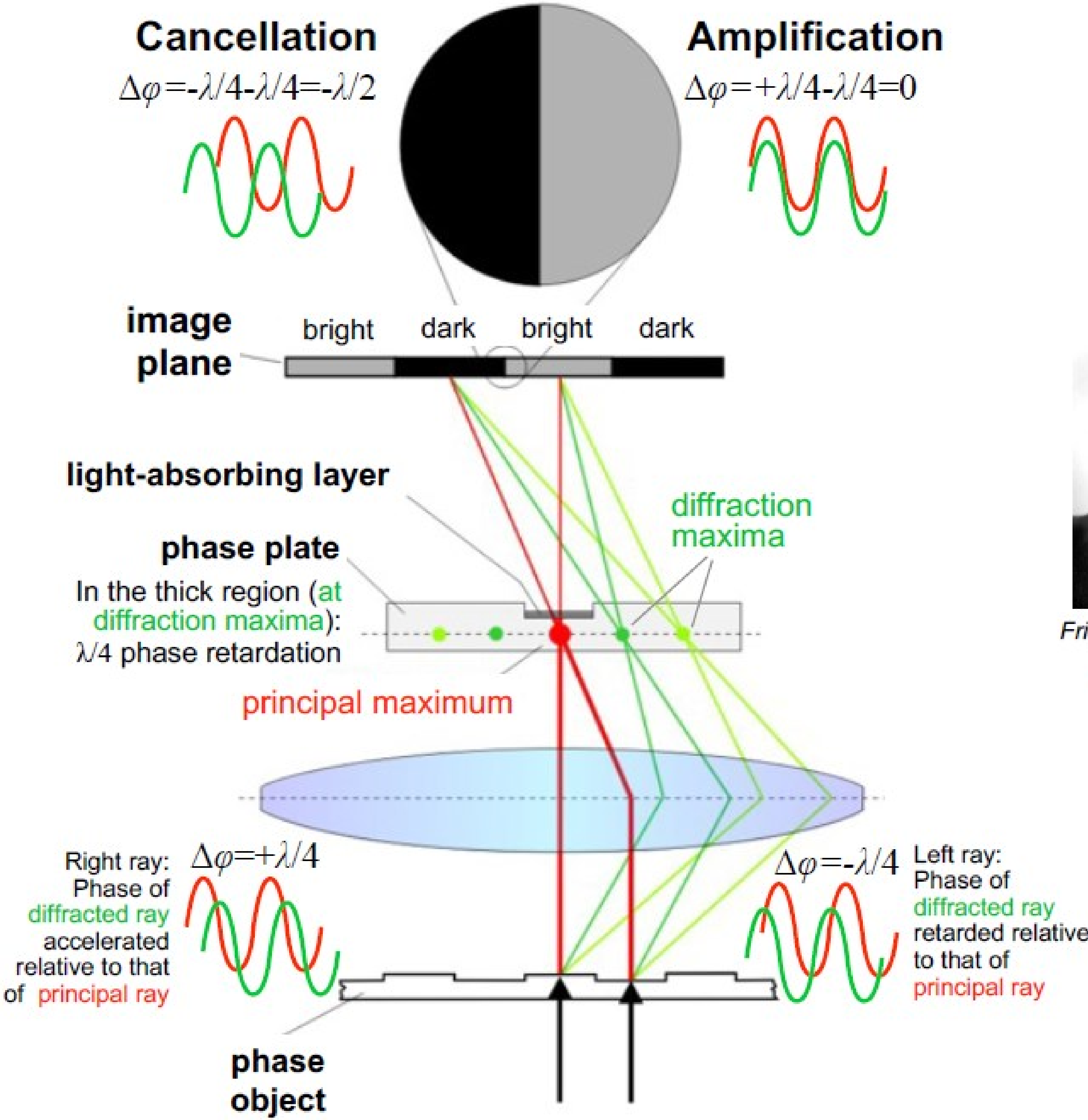
$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi_0)$$
$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0 \cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \phi_0)$$

At any point of the observation we have to add all of the incoming sine waves, and that gives the net value of **E** and **B**.

Remember: incoherent waves add up to practically 0, while coherent ones can add up from 0 to a maximum, depending on the phase difference.

Demonstration of the Abbe's principle with a slit+stage micrometer under the microscope





Frits Zernike (1888-1966)
Nobel-prize

Applications:

Optical grating can separate the different frequencies -> recording of a spectrum